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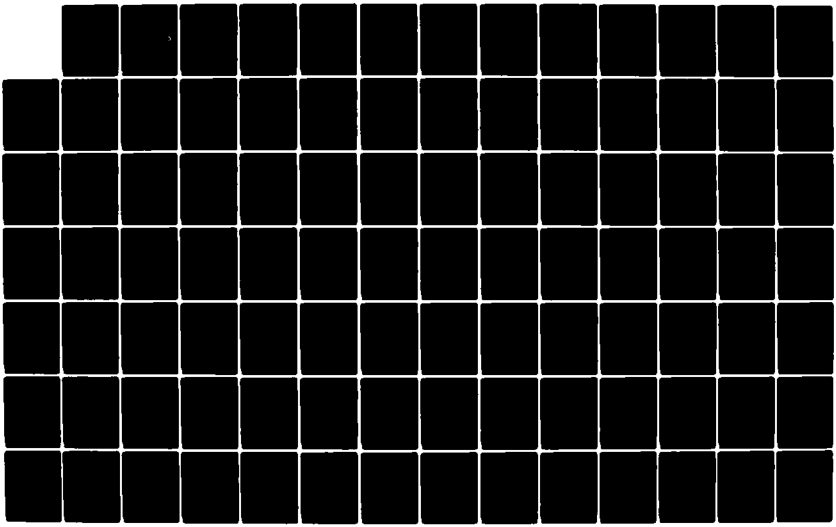

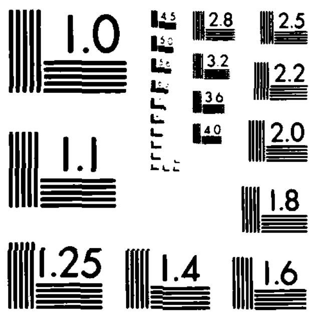

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Abstract

of

APPROXIMATE SOLUTION FOR A LAMINAR FLOW OVER A FLAT PLATE

WITH SUCTION AND PRESSURE GRADIENT

by

George Francesco D'Amore

Statement of Problem

This thesis derives and analyzes an approximate velocity solution that represents a suction flow over a flat plate. The solution is derived from the momentum integral equation in terms of the suction and pressure gradient parameters. These parameters are adjusted until flow separation is achieved. Further analysis is done with displacement thickness, momentum thickness, and friction. Other proven solutions are compared with this analysis.

Sources of Data

The information was collected through a literature review of journals, books, and goverment publications. This included the original studies dated from 1945 to present. The solutions derived were analyzed through computer graphics and data output.

Conclusions Reached

The approximate solution is in good agreement with the Blasius solution for zero suction and zero pressure gradient. Increasing suction velocity prevents or delays flow separation, and permits a flow to withstand higher adverse pressure gradient. The solution is limited to a suction parameter equal to -2 and consequently an optimum suction velocity is found. Suction velocity decreases the boundary layer thickness, and adverse pressure gradient increases the boundary layer thickness.

Committee Chair's Signature of Approval James Bergquam

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AUTHOR: George Francesco D'Amore

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APPROXIMATE SOLUTION FOR A LAMINAR FLOW OVER A FLAT PLATE WITH SUCTION AND PRESSURE GRADIENT

George Francesco D'Amore

Captain, USAF

B.S., Wichita State University

THESIS

Submitted in partial satisfaction of
the requirements for the degree of

MASTER OF SCIENCE

in

MECHANICAL ENGINEERING

at

CALIFORNIA STATE UNIVERSITY, SACRAMENTO

Summer
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A Thesis

by

George Francesco D'Amore

Approved by:

James B. Bergquam, Advisor
James B. Bergquam

Praveen K. Amar, Second Reader
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Department of Mechanical Engineering

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Committee Chair's Signature of Approval James Bergquam

Table of Contents

Symbols

Symbol	Description
C_D	Coefficient of Drag
a, b, c, d	Coefficients of 4th Order Polynomial
M	Suction Parameter, $v_o \delta / \nu$
N	Pressure Gradient Parameter, $\delta^2 dU_\infty / \nu dx$
p	Constant Multiplied by N
P	Pressure
t	Time
u	Velocity Component in X-direction inside Boundary Layer
v	Velocity Component in Y-direction inside Boundary Layer
w	Velocity Component in Z-direction inside Boundary Layer
v_o	Normal Velocity at the Wall ($v_o < 0$ for suction)
U_∞	Free Stream Velocity outside Boundary Layer
u'	Dimensionless Velocity in X-direction
v'	Dimensionless Velocity in Y-direction
x'	Dimensionless Length of X
y'	Dimensionless Length of Y
p'	Dimensionless Pressure

δ	Boundary Layer Thickness, $\sqrt{\nu x / U_\infty}$
δ_1	Displacement Thickness
δ_2	Momentum Thickness
η	Non-dimensional Length, y/δ
μ	Absolute Viscosity
ν	Kinematic Viscosity
ρ	Density
τ_o	Shear Stress at the Surface

Figures

Tables

Chapter 1

Introduction

This thesis derives an approximate solution to study the effects of suction for flow on a flat plate. Most available solutions are in terms of either pressure gradient only, as in the solution proposed by Pohlhausen, or in terms of suction velocity alone. One purpose of this study is to derive a velocity profile that is easy to adjust for changing suction and pressure gradient parameters. Another objective of this thesis is to analyze the newly derived velocity profile and to observe the effects that suction and pressure gradient have on the approximate velocity profile.

A dimensionless velocity profile is solved in terms of suction and pressure gradient parameters. Pohlhausen assumed a fourth order polynomial which is also assumed in this thesis; however, different boundary conditions are used. The boundary conditions proposed by Torda are used in Chapter 3 along with the proposed fourth order polynomial to solve the approximate velocity profile used throughout this thesis. The approximate velocity profile is used to analyze the effects of suction and pressure gradient by observing displacement thickness, momentum thickness, and shear stress

at the wall.

The flow being analyzed is limited to a steady, two-dimensional, incompressible, laminar flow. Pressure gradient is limited to the point of separation. Another purpose of this thesis is to observe the movement of separation profile as suction is applied to the derived velocity profile equation. The entire solution is in dimensionless form where u/U_∞ is restricted to remain between the limits of zero and one. If u/U_∞ were to exceed the set limits, the flow could not be considered a steady flow.

In Chapter 2 the momentum equation is derived with the suction term; this equation then enables the critical suction velocity to be computed. The suction velocity needed to keep a flow from separating is derived in Chaper 4 by use of the momentum equation. The derived equations from Chapter 3 (dimensionless velocity profile, displacement thickness, momentum thickness, and wall shear stress) are compared with the solution of Blasius in Chapter 4 and analyzed for the different suction and pressure gradient parameters which are then tabulated and shown graphically.

CHAPTER 2

Derivation of Boundary Layer Equations with Suction

2.1 Derivation of the Boundary Layer Equations

The Navier-Stokes equations have been the primary equations for solving fluid flow problems. Prandtl simplified the Navier-Stokes equations by eliminating some of the terms to develop the boundary layer equations.

The complete Navier-Stokes equations for two-dimensional flow are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1\partial P}{\rho\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1\partial P}{\rho\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{2}$$

The boundary conditions to be satisfied once the boundary layer equation is derived will be:

at $y = 0$; $u = 0$, $v = 0$

at $y = \delta$; $u = U_\infty$

Since the flow on the flat plate is a steady flow only in the two dimensions of x and y, the Navier-Stokes equations can be simplified because:

$$w\frac{\partial u}{\partial z} = w\frac{\partial v}{\partial z} = 0 \quad ; \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0 \quad ; \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$$

The Navier-Stokes equations can be reduced to boundary layer form by estimating the order of magnitude of each quantity as proposed by Prandtl. By letting the free stream velocity U_∞ be the unit of magnitude for velocity, the terms of the boundary layer equations can be put into non-dimensional form.[1] The boundary layer thickness δ is assumed to be very small compared with x:

$$u' = \frac{u}{U_\infty} \quad ; \quad v' = \frac{v}{U_\infty} \quad ; \quad p' = \frac{p}{\rho U_\infty{}^2}$$

$$x' = \frac{x}{L} \quad ; \quad y' = \frac{y}{L} \quad ; \quad \delta' = \frac{\delta}{L}$$

where L is the boundary length.

For the order of magnitude:

$u' = 1$ since u changes from 0 to U_∞

$v' = \delta$ since y changes from 0 to δ

$x' = 1$

$y' = \delta$

$Re = 1/\delta^2$

Both u and v are functions of x and y.

As a result of the preceding assumptions, the order of magnitude of other terms can be determined as follows:

$$\frac{\partial u'}{\partial x'} = 1 \quad ; \quad \frac{\partial^2 u'}{\partial x'^2} = 1$$

---

1 Superscripts refer to endnotes on page 78.

$$\frac{\partial u'}{\partial y'} = \frac{1}{\delta} \quad ; \quad \frac{\partial^2 u'}{\partial y'^2} = \frac{1}{\delta \cdot \delta} = \frac{1}{\delta^2}$$

$$\frac{\partial v'}{\partial x'} = \delta \quad ; \quad \frac{\partial^2 v'}{\partial x'^2} = \frac{\delta}{1 \cdot 1} = \delta$$

$$\frac{\partial v'}{\partial y'} = \frac{\delta}{\delta} = 1 \quad ; \quad \frac{\partial^2 v'}{\partial y'^2} = \frac{\delta}{\delta \cdot \delta} = \frac{1}{\delta}$$

With these newly derived terms, Equations 1 and 2 can be rewritten into non-dimensional form as follows:

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = -\frac{\partial P'}{\partial x'} + \frac{1}{Re}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) \tag{3}$$

$$1 \cdot 1 \qquad \delta \cdot \frac{1}{\delta} \qquad\qquad \delta^2\left(1 \qquad \frac{1}{\delta^2}\right)$$

For the y direction:

$$u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'} = -\frac{\partial P'}{\partial y'} + \frac{1}{Re}\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) \tag{4}$$

$$1 \cdot \delta \qquad \delta \cdot 1 \qquad\qquad \delta^2\left(\delta \qquad \frac{1}{\delta}\right)$$

The order of magnitude of the inertial terms equals the order of magnitude of the viscous terms in Equations 3 and 4. Since 1 is smaller than $1/\delta^2$ in Equation 3, $\partial^2 u' / \partial x'^2$ can be neglected. The order of magnitude of $1/Re$ is δ^2.[2] As a result Equation 3 becomes the x-direction of Prandtl's original boundary-layer equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Applying the same analysis of Equation 3 to Equation 4, shows that $(1/\rho)\partial P/\partial y = \delta$, which implies that $(1/\rho)\partial P/\partial y = 0$ since δ is very small. The final forms for the boundary layer equations are:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (5)$$

$$\frac{\partial P}{\partial y} = 0 \qquad (6)$$

The continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7)$$

Conditions outside the boundary-layer are:

$u = U_\infty$; $v = 0$; $\partial u/\partial y = \partial^2 u/\partial y^2 = 0$; $\nu = 0$

Applying these conditions to the x-direction momentum equation, Equation 5, leads to:

$U_\infty(dU_\infty/dx) = -(1/\rho)dP/dx$

Now the revised version of Equation 5 is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty\frac{dU_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (8)$$

The boundary conditions are:

at $y = 0$; $u = 0$, $v = 0$

at $y = \delta$; $u = U_\infty$

The Prandtl equations describe the flow of fluid without the effect of pressure gradient or suction, whereas Pohlhausen solved the problem of boundary layer flow with variable pressure gradient, but without suction. Pohlhausen's approximate method is based on the momentum equation. The momentum equation is derived from Prandtl's boundary layer equation and is often referred to as the von Karman momentum-integral equation.

2.2 Derivation of the Momentum-Integral Equation

Using Prandtl's boundary layer equation, Equation 8, and integrating it with respect to y, from $y = 0$ at the wall of the surface to $y = \infty$, yields:

$$\int_0^\infty \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - U_\infty \frac{dU_\infty}{dx} \right) dy = \nu \frac{\partial u}{\partial y} \Bigg|_0^\infty \tag{9}$$

The fluid flow also satisfies the equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By separating the terms algebraically and integrating each term into the continuity equation, suction v_o can be introduced into the equation as follows:

$$\frac{\partial v}{\partial y} = - \frac{\partial u}{\partial x}$$

Integrating each term with respect to y yields:

$$\int_0^y \frac{\partial v}{\partial y} dy = -\int_0^y \frac{\partial u}{\partial x} dy$$

$$v\Big|_y - v\Big|_{y=0} = -\int_0^y \frac{\partial u}{\partial x} dy$$

Therefore, the normal velocity component $v(y)$ is:

$$v(y) = v(0) - \int_0^y \frac{\partial u}{\partial x} dy \tag{10}$$

and $v(0)$ is the velocity at the wall, or as in this case, $v(0)$ is the suction term. Pohlhausen did not apply suction and consequently $v(0) = 0$.

In Pohlhausen's approximate solution the normal velocity term used in the momentum-integral equation is:

$$v(y) = -\int_0^y \frac{\partial u}{\partial x} dy \tag{11}$$

The shear stress is known to be $\tau_o = -\mu(\partial u/\partial y)$ where τ_o is shear stress at the wall. To agree with the momentum equation ρ and μ are substituted so that:

$$\tau_o = -\rho\nu \frac{\partial u}{\partial y} \tag{12}$$

Equation 9 is rewritten with the substitution of the new terms, Equations 11 and 12. The variable, y, is the distance from the surface to a point in the free stream identified by $y = h$.[3]

$$\int_0^h \left(u\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\int_0^y \frac{\partial u}{\partial y} dy - U_\infty \frac{dU_\infty}{dx} \right) dy = -\frac{\tau_o}{\rho} \tag{13}$$

Integration by parts is used to resolve the normal velocity integral. The rule for integration by parts is:

$$\int_0^h m\,dn = mn\Big|_0^h - \int_0^h n\,dm$$

To define terms, let:

$$m = \int_0^y \frac{\partial u}{\partial x}dy \quad , \; dn = \frac{\partial u}{\partial y}dy \quad , \; dm = \frac{\partial u}{\partial x}dy \quad , \; n = u$$

Combining and substituting the terms gives:

$$\int_0^h \left(\frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x}dy \right) dy = u\int_0^y \frac{\partial u}{\partial x}dy\Big|_0^h - \int_0^h u\frac{\partial u}{\partial x}dy$$

at $y = 0$, $u = 0$

at $y = h$, $u = U_\infty$

Therefore the right side of the above equation becomes:

$$U_\infty \int_0^h \frac{\partial u}{\partial x}dy - \int_0^h u\frac{\partial u}{\partial x}dy \qquad (14)$$

Substituting Expression 14 into Equation 13 develops the momentum equation into:[4]

$$\int_0^h u\frac{\partial u}{\partial x}dy - U_\infty \int_0^h \frac{\partial u}{\partial x}dy + \int_0^h u\frac{\partial u}{\partial x}dy - \int_0^h U_\infty \frac{dU_\infty}{dx}dy = -\frac{\tau_0}{\rho}$$

which can be condensed to:

$$\int_0^h \left(2u\frac{\partial u}{\partial x}dy - U_\infty \frac{\partial u}{\partial x} - U_\infty \frac{dU_\infty}{dx} \right) dy = -\frac{\tau_o}{\rho} \qquad (15)$$

Since $U_\infty = f(x)$ and $u = f(x,y)$, and:

$$\frac{\partial}{\partial x}(uU_\infty) = u\frac{dU_\infty}{dx} + U_\infty \frac{\partial u}{\partial x}$$

then, Equation 14 can be rewritten as:

$$\int_0^h \left(- \frac{\partial}{\partial x}(u)^2 + \frac{\partial}{\partial x}(uU_\infty) - u\frac{dU_\infty}{dx} + U_\infty\frac{dU_\infty}{dx} \right) dy = \frac{\tau_0}{\rho}$$

The above equation is condensed into terms that are more recognizable as displacement thickness and momentum thickness:

$$\int_0^h \left(\frac{\partial}{\partial x}(uU_\infty - u^2) + \frac{dU_\infty}{dx}(U_\infty - u) \right) dy = \frac{\tau_0}{\rho} \qquad (16)$$

By definition δ_1 = displacement thickness, where:

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U_\infty} \right) dy$$

After factoring U_∞ out,

$$\delta_1 U_\infty = \int_0^\infty (U_\infty - u)\, dy$$

Displacement thickness is the measure of the necessary displacement of the potential flow from the surface to offset the formation of the increasing size of the boundary layer as distance along the flat plate increases. The stream lines for the potential flow outside the boundary layer are deflected a distance δ_1. The deflection is due to the effects of friction near the flat plate. Figure 2.1 shows the definition of displacement thickness where area A and area B are equal. Also, by definition δ_2 is equal to momentum thickness, where:

$$\delta_2 = \int_0^\infty \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty} \right) dy$$

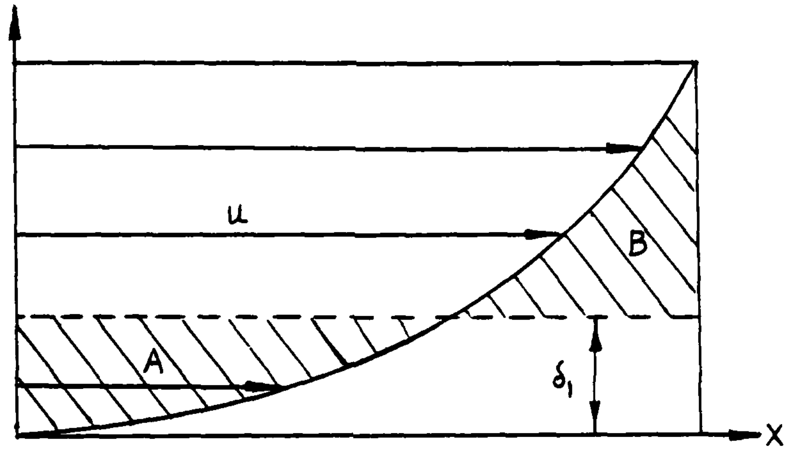

Figure 2.1

Displacement of Potential Flow

The momentum thickness is defined to be the measure of the loss of momentum in the boundary layer. This is quite similar to displacement thickness where potential flow is measured.

The expression $\partial/\partial x$, can be taken outside the integral in Equation 16. The limit $(0, \infty)$ is independent of x and the general momentum equation becomes:

$$\frac{d}{dx}[\delta_2 U_\infty^2] + \frac{dU_\infty}{dx}[\delta_1 U_\infty] = \frac{\tau_o}{\rho}$$

The general momentum equation is further expanded to:

$$\frac{d\delta_2}{dx} + \frac{\delta_2}{U_\infty}\left(\frac{dU_\infty}{dx}\right)\left(2 + \frac{\delta_1}{\delta_2}\right) = \frac{\tau_o}{\rho U_\infty^2}$$

Pohlhausen used this equation to solve his flow problem with variable pressure gradient. In this thesis the same problem

is solved but with suction. This is done by applying the suction term to the momentum equation. As seen previously, Equation 10 is:

$$v(y) = v(0) - \int_0^y \frac{\partial u}{\partial x} dy$$

which is then substituted into Equation 9 to solve for the momentum equation with suction. It is solved the same way as the momentum equation without suction. The final solution is:

$$\frac{d}{dx}[\delta_2 U_\infty^2] + \delta_1 U_\infty \frac{dU_\infty}{dx} + U_\infty v_o = \frac{\tau_o}{\rho}$$

which is expanded to:

$$U_\infty^2 \frac{d\delta_2}{dx} + (2\delta_2 + \delta_1) U_\infty \frac{dU_\infty}{dx} + U_\infty v_o = \frac{\tau_o}{\rho}$$

Therefore the momentum equation with suction is:

$$\frac{d\delta_2}{dx} + \frac{\delta_2}{U_\infty}\left(\frac{dU_\infty}{dx}\right)\left(2 + \frac{\delta_1}{\delta_2}\right) + \frac{v_o}{U_\infty} = \frac{\tau_o}{\rho U_\infty^2} \tag{17}$$

The term v_o is the suction velocity (or blowing) through the wall. By using the momentum equation, Equation 11, the critical suction required to keep the flow from separation is found. To have separation, adverse pressure gradient is necessary. Chapter 3 derives an approximate velocity profile that is used in the momentum equation in order to find the suction velocity necessary to prevent separation. The velocity profile is derived in terms of suction and pressure gradient.

CHAPTER 3

Approximate Solution with Suction

To show the effects of suction on a boundary layer a method similar to Pohlhausen's approximate solution is used. However, according to Torda, Pohlhausen's boundary conditions created a problem when used with his fourth order polynomial. A certain suction velocity value caused the equation to approach infinity which is not realistic in actual flow. Torda explained that this problem would be avoided by changing the boundary conditions and the boundary layer equation.[6]

3.1 <u>Derivation of the Non-dimensional Velocity Equation</u>

Pohlhausen's non-dimensional velocity polynomial is:

$$\frac{u}{U_\infty} = a\eta + b\eta^2 + c\eta^3 + d\eta^4 \qquad (1)$$

where η is a non-dimensional term defined as y/δ. Appendix A explains in detail how Pohlhausen derived the velocity, displacement thickness, and momentum thickness equations. The boundary conditions used by Pohlhausen are not the same as those proposed by Torda. Pohlhausen's approximate solution shows the effects of favorable and adverse pressure gradient on a flow. Adding suction to the flow

changes the reaction of the flow. Suction from a wall affects the flow and separation for different values of adverse pressure. The diverse changes of a flow due to pressure gradient and suction are shown in Chapter 4.

The boundary conditions used are:

at $y = 0$ $(\eta = 0)$, $u = 0$, $v_o = 0$

at $y = \delta$ $(\eta = 1)$, $u = U_\infty$, $\partial u/\partial y = \partial^2 u/\partial y^2 = 0$

The boundary layer equation from Chapter 2 is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right) + \nu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

Outside the boundary layer u is constant and equal to U_∞ and $\partial u/\partial x = 0$. Also, v is zero causing Equation 2 to become:

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = U_\infty\frac{dU_\infty}{dx} \qquad (3)$$

Substituting Equation 3 into the boundary layer equation, Equation 2, yields:

$$v_o\frac{\partial u}{\partial y} = U_\infty\frac{dU_\infty}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (4)$$

The derivative of Equation 4 yields another boundary equation at $y = 0$:

$$v_o\frac{\partial^2 u}{\partial y^2} = \nu\left(\frac{\partial^3 u}{\partial y^3}\right)$$

The first and second derivatives of Pohlhausen's polynomial, Equation 1, yield:

$$\frac{\partial u}{\partial \eta} = a + 2b\eta + 3c\eta^2 + 4d\eta^3 \qquad (5)$$

$$\frac{\partial^2 u}{\partial \eta^2} = 2b + 6c\eta + 12d\eta^2 \qquad (6)$$

At $y = \delta$ ($\eta = 1$), Equations 5 and 6 equal zero along the boundary layer. Velocity u is a constant U_∞ outside the boundary layer flow and is a function of y below the boundary layer. Figure 3.1 shows the effects of a velocity profile at the plate surface and at the boundary layer thickness. The profile changes when suction is applied.

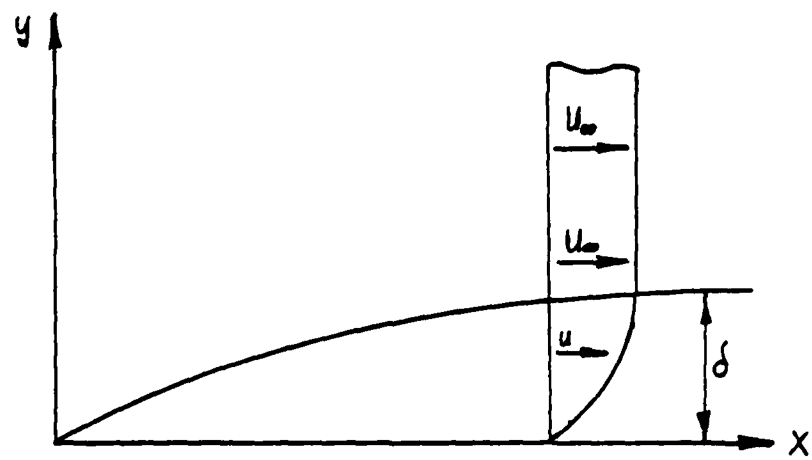

Figure 3.1

Velocity Flow Profile

Substituting the above boundary conditions into Equations 5 and 6 respectively yields:

$$\frac{\partial u}{\partial y} = 0 = a + b + 3c + 4d \tag{7}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 = 2b + 6c + 12d \tag{8}$$

Since $u = U_\infty$ at $y = \delta$ ($\eta = 1$), Equation 1 becomes:

$$\frac{u}{U_\infty} = 1 = a + b + c + d \tag{9}$$

At the wall, where $y = 0$ ($\eta = 0$), the derivative of Equation 6 becomes:

$$\frac{\partial^3 u}{\partial y^3} = 6c + 24d\eta$$

$$\frac{\partial^3 u}{\partial y^3} = 6c \tag{10}$$

Applying the same conditions as those used for Equation 10 to Equations 5 and 6 yields:

$$\frac{\partial u}{\partial y} = a \tag{11}$$

$$\frac{\partial^2 u}{\partial y^2} = 2b \tag{12}$$

The terms in Equation 4 and its derivative are kept in non-dimensional form where $\eta = y/\delta$; thus $\partial u/\partial y$, $\partial^2 u/\partial y^2$, and $\partial^3 u/\partial y^3$ can be rewritten using the chain rule:

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\ \frac{\partial \eta}{\partial y} \quad = \frac{\partial u}{\partial \eta}\ \frac{1}{\delta} \tag{13}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial \eta}\ \frac{1}{\delta}\right) = \frac{1}{\delta}\ \frac{\partial^2 u}{\partial \eta \partial y} = \frac{1}{\delta}\ \frac{\partial^2 u}{\partial \eta^2}\ \frac{\partial \eta}{\partial y}$$

therefore:

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\delta^2}\ \frac{\partial^2 u}{\partial \eta^2} \tag{14}$$

$$\frac{\partial^3 u}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{1}{\delta^2}\ \frac{\partial^2 u}{\partial \eta^2}\right) = \frac{1}{\delta^2}\ \frac{\partial^3 u}{\partial \eta^2 \partial y} = \frac{1}{\delta^2}\ \frac{\partial^3 u}{\partial \eta^3}\ \frac{\partial \eta}{\partial y}$$

therefore:

$$\frac{\partial^3 u}{\partial y^3} = \frac{1}{\delta^3}\ \frac{\partial^3 u}{\partial \eta^3} \tag{15}$$

Substituting Equations 11 and 12 into Equation 4 solves the equation in terms the flow of fluid without the effects of pressure gradient or suction, whereas Pohlhausen solved the problem of boundary layer flow with variable pressure gradient, but without suction. Inserting Pohlhausen's approximate method results of Equations 7 and 8, and the expressions of Equations 13 and 14 into Equation 4 gives:

$$\frac{v_0}{\delta}(a) = U_\infty \frac{dU_\infty}{dx} + \frac{\nu}{\delta^2}(2b) \tag{16}$$

The same procedure is used with the derivative of Equation 4 for Equations 8, 10, 14, and 15.

$$\frac{v_0}{\delta^2}(2b) = \frac{\nu}{\delta^3}(6c) \tag{17}$$

The objective now is to combine and rearrange the equations formed so that the coefficients of Equation 1 can be solved. The equations to be solved need to be in terms of suction and pressure gradient. According to Pohlhausen the dimensionless parameter, $N = (\delta^2/\nu)dU_\infty/dx$, will account for the pressure change that is involved in fluid flow. Since $dP/dx = (-\rho U_\infty)dU_\infty/dx$, the dimensionless parameter $N = (-\delta^2/(\mu U_\infty))dP/dx$.[7] Consequently U_∞ is affected by the pressure change as the flow proceeds.

Iglish theorized the development of suction flow without pressure gradient, and the term he used to account for suction is $M = v_0\delta/\nu$.[8] The terms M and N will be incorporated into the newly derived equations of: u/U_∞, shear, displacement thickness δ_1, and momentum thickness δ_2. To solve for the terms a, b, c, and d, multiply Equation 16 by $3/\delta$.

$$\frac{v_0}{\delta^2}(3a) = 3\frac{U_\infty}{\delta}\frac{dU_\infty}{dx} + \frac{\nu}{\delta^3}(6b) \tag{18}$$

Summing Equations 17 and 18 yields:

$$\frac{v_0}{\delta^2}(3a + 2b) = 3\frac{U_\infty}{\delta}\frac{dU_\infty}{dx} + \frac{\nu}{\delta^3}6(b + c) \tag{19}$$

To put Equation 19 in terms of the coefficients b and c, multiply Equation 9 by four and subtract Equation 7 from it.

This will yield:

$$4 - c = 3a + 2b$$

which is substituted into Equation 19 giving:

$$\frac{v_o}{\delta^2}(4 - c) = 3\frac{U_\infty}{\delta}\frac{dU_\infty}{dx} + \frac{\nu}{\delta^3}6(b + c)$$

From Equation 17, $b = 3\nu c / v_o \delta$. After substituting coefficient b into the above equation, coefficient c is solved as:

$$c = \frac{-\dfrac{3}{\delta}\dfrac{U_\infty dU_\infty}{dx} + \dfrac{4v_o}{\delta^2}}{\dfrac{18\nu^2}{v_o\delta^4} + \dfrac{6\nu}{\delta^3} + \dfrac{v_o}{\delta^2}} \tag{20}$$

Multiplying the numerator and denominator of Equation 20 by $v_o\delta^4 / \nu^2$ will leave the coefficient c in terms of the suction parameter M and the pressure gradient parameter N.

$$c = \frac{\left(-\dfrac{3}{\delta}\dfrac{U_\infty dU_\infty}{dx} + \dfrac{4v_o}{\delta^2}\right)\left(\dfrac{v_o\delta^4}{\nu^2}\right)}{\left(\dfrac{18\nu^2}{v_o\delta^4} + \dfrac{6\nu}{\delta^3} + \dfrac{v_o}{\delta^2}\right)\left(\dfrac{v_o\delta^4}{\nu^2}\right)}$$

Therefore:

$$c = \frac{-\dfrac{3U_\infty dU_\infty}{dx}\dfrac{v_o\delta^3}{\nu^2} + \dfrac{4v_o^2\delta^2}{\nu^2}}{18 + \dfrac{6v_o\delta}{\nu} + \dfrac{v_o^2\delta^2}{\nu^2}}$$

To convert coefficient c into terms of M and N, write:

$$c = \frac{-3MN + 4M^2}{(18 + 6M + M^2)} \tag{21}$$

Coefficient b is $3\nu c / v_o \delta = 3c / M$. Substituting Equation 21 into the term for b gives:

$$b = \frac{3[-3N + 4M]}{(18 + 6M + M^2)} \tag{22}$$

From the expression $4 - c = 3a + b$, coefficient a is solved as:

$$a = \frac{4 - 2b - c}{3}$$

Substituting Equations 21 and 22 into the preceding equation for coefficient a yields:

$$a = \frac{24 + 6N + MN}{(18 + 6M + M^2)} \tag{23}$$

To obtain the coefficient d, Equation 7 is used:

$$0 = a + 2b + 3c + 4d \tag{7}$$

Again substituting Equations 21 and 22 along with Equation 23 into Equation 7 and solving for the coefficient d yields:

$$d = \frac{-6 + 3N + 2MN - 6M - 3M^2}{(18 + 6M + M^2)} \tag{24}$$

Equation 1 is now written in its full and complete form. To help alleviate the congestion of terms let $D = 18 + 6M + M^2$.

$$u/U_\infty = (24\eta + 12M\eta^2 + 4M^2\eta^3 + (6 - 6M - 3M^2)\eta^4) / D$$

$$+ (N((6 + M)\eta - 9\eta^2 - 3M\eta^3 + (3 + 2M)\eta^4)) / D \qquad (25)$$

Equation 25 expresses the flow for various suction and pressure gradient parameters. For various values of M and N, the different separation locations are found. Whereas, Pohlhausen's equation, derived in Appendix A, shows flow reactions that are dependent only on variable pressure gradients.

3.2 Derivation of the Shear Stress Equation

The shear stress at the wall where the flow begins separation is zero. Applying the same principle used by Pohlhausen and shown in Appendix A leads to:

$$\left(\frac{\partial u}{\partial y}\right) = a \quad \text{and} \quad \tau_0 = \left(\frac{\partial u}{\partial y}\right)_0$$

To keep the equations in non-dimensional form, Equation 13 is used.

$$\left(\frac{\partial u}{\partial \eta}\right)\left(\frac{\partial \eta}{\partial y}\right) = \left(\frac{\partial u}{\partial \eta}\right)\left(\frac{1}{\delta}\right)\Bigg|_{\eta=0} = a$$

After substituting Equation 23 into the above expressions, the shear stress equation is derived in terms of suction and pressure gradient as:

$$\frac{\tau_0 \delta}{\mu U_\infty} = \frac{24 + 6N + MN}{18 + 6M + M^2} \qquad (26)$$

3.3 Derivation of the Displacement Thickness Equation

Solving for displacement thickness δ_1 is a tedious process because of the inherent multiple computations of approximate methods. By definition, we know that:

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy$$

Keeping δ_1 non-dimensional and using $\eta = y/\delta$, gives:

$$\delta_1 = \int_0^1 \delta \left(1 - \frac{u}{U_\infty}\right) d\eta \quad \text{where } d\eta \text{ has the limits 0 to 1.}$$

Substituting Equation 1 into the above expression for δ_1/δ yields:

$$\frac{\delta_1}{\delta} = \int_0^1 (1 - (a\eta + b\eta^2 + c\eta^3 + d\eta^4))d\eta$$

Integrating δ_1/δ gives:

$$\frac{\delta_1}{\delta} = \left(\eta - \frac{a\eta^2}{2} - \frac{b\eta^3}{3} - \frac{c\eta^4}{4} - \frac{d\eta^5}{5}\right)\Bigg|_0^1 \tag{27}$$

After expanding the above equation and combining common terms, Equation 27 becomes:

$$\frac{\delta_1}{\delta} = \frac{1}{60}(60 - 30a - 20b - 15c - 12d) \tag{28}$$

Using Equation 23 the term 30a, yields:

$$30a = (720 + 180N + 30MN) / D$$

Using Equation 22 in the term 20b, gives:

$$20b = (-180N + 240M) / D$$

With the aid of Equation 21 the term 15c yields:

$15c = (-45MN + 60M^2) / D$

and, the last term, 12d, is found with Equation 24.

$12d = (-72 + 36N + 24MN - 72M - 36M^2) / D$

Taking these four derived terms and substituting them back into Equation 28 yields the non-dimensional displacement thickness:

$$\frac{\delta_1}{\delta} = \frac{1}{60D}(432 - 36N - 9MN + 192M + 36M^2) \tag{29}$$

where D is $(18 + 6M + M^2)$.

3.4 Derivation of the Momentum Thickness Equation

The momentum thickness δ_2 is defined as:

$$\delta_2 = \int_0^{\infty} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Writing δ_2 into non-dimensional form, δ_2 / δ becomes:

$$\frac{\delta_2}{\delta} = \int_0^1 \left(\frac{u}{U_\infty} - \left(\frac{u}{U_\infty}\right)^2\right) d\eta \tag{30}$$

Equation 30 is expanded to decrease the mathematical computations and is rewritten as:

$$\frac{\delta_2}{\delta} = \int_0^1 \frac{u}{U_\infty} d\eta - \int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta \tag{31}$$

The equation, $\int_0^1 (u / U_\infty) d\eta$, has been worked out already and is equivalent to (1 - Equation 29) giving:

$$\int_0^1 \frac{u}{U_\infty} d\eta = (648 + 36N + 9MN + 168M + 24M^2) / 60D \tag{32}$$

The second term of Equation 31 is much more involved than the first term since Equation 25 is squared. Equation 1 could have been assumed to be a second or third order polynomial to help ease the numerous calculations, however accuracy would have been compromised. Schlichting stated that assuming a polynomial with more than four coefficients would not increase the accuracy of the approximate solution.[9] It should be noted that Schlichting was referring to Pohlhausen's solution of flow. Pohlhausen's solution does not involve suction. Equation 25 does include suction along with pressure gradient.

In reference to Equation 1 and the second term of Equation 31, $(u/U_\infty)^2$, becomes:

$$\left(\frac{u}{U_\infty}\right)^2 = (a\eta)^2 + ab\eta^3 + ac\eta^4 + ad\eta^5 + ba^3 + b^2\eta^4 + bc\eta^5$$
$$+ bd\eta^6 + ca\eta^4 + cb\eta^5 + c^2\eta^6 + cd\eta^7 + da\eta^5$$
$$+ db\eta^6 + dc\eta^7 + d^2\eta^8$$

Collecting terms yields:

$$\left(\frac{u}{U_\infty}\right)^2 = a^2\eta^2 + 2ab\eta^3 + 2ac\eta^4 + 2ad\eta^5 + b^2\eta^4 + 2bc\eta^5$$
$$+ 2bd\eta^6 + c^2\eta^6 + 2cd\eta^7 + d^2\eta^8$$

Integrating the above equation as described in Equation 31 gives:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = \left(\frac{a^2\eta^3}{3} + \frac{ab\eta^4}{2} + \frac{2ac\eta^5}{5} + \frac{ad\eta^6}{3} + \frac{b^2\eta^5}{5} + \frac{bc\eta^6}{3}\right.$$

$$\left. + \frac{2bd\eta^7}{7} + \frac{c^2\eta^7}{7} + \frac{cd\eta^8}{4} + \frac{d^2\eta^9}{9}\right)\Bigg|_0^1$$

Then after applying the limits, the above equation becomes:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = \frac{a^3}{3} + \frac{ab}{2} + \frac{2ac}{5} + \frac{ad}{3} + \frac{b^2}{5} + \frac{bc}{3}$$

$$+ \frac{2bd}{7} + \frac{c^2}{7} + \frac{cd}{4} + \frac{d^2}{9}$$

Collecting terms and simplifying them gives:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = (1/1260)(420a^2 + 630ab + 504ac$$

$$+ 420ad + 252b^2 + 420bc + 360bd$$

$$+ 180c^2 + 315cd + 140d^2) \tag{33}$$

With further collection and simplification of terms Equation 33 becomes:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = (1/1260)[\, a(420a + 630b + 504c)$$

$$+ b(420c + 252b) + c(180c)$$

$$+ d(140d + 420a + 360b + 315c)] \tag{34}$$

The terms a, b, c, and d which are Equations 23, 22, 21, and 24 respectively are then substituted into Equation 34. This task is very involved and quite tedious. The work showing the derivation of δ_2/δ is further shown in Appendix B. From Appendix B the momentum thickness equation is:

$$\frac{\delta_2}{\delta} = (58464 - 729N + 948MN + 40032M + 12816M^2 - 612N^2 - 303MN^2 + 624M^2N - 38M^2N^2 + 78M^3N + 1872M^3 + 144M^4) / (1260(18 + 6M + M^2)^2) \tag{35}$$

CHAPTER 4

Analysis of Approximate Solution with Suction

4.1 Comparison of Approximate Solution with Blasius

The previous two chapters went into detail deriving the necessary equations to analyze suction flow over a flat plate. It was also necessary to involve the effects of pressure gradient which adjusts the flow whether suction is applied or not. The equations in Chapter 3 had two parameters that were variable--suction and pressure gradient.

Some of the equations are very long and involved. To study the accuracy of this approximate solution for suction flow, it is necessary to check the approximate method against proven data given by Blasius and other authors.

Blasius' data is based on no pressure gradient and no suction. To compare the thesis solution with the Blasius solution the integral momentum equation derived in Chapter 2 is used:

$$\frac{d\delta_2}{dx} + \frac{\delta_2}{U_\infty}\left(\frac{dU_\infty}{dx}\right)\left(2 + \frac{\delta_1}{\delta_2}\right) + \frac{v_o}{U_\infty} = \frac{\tau_o}{\rho U_\infty^2}$$

Using the same assumptions as Blasius, where there is no suction or pressure gradient to contend with, the integral momentum equation is reduced to a more simple form. For the

problem considered, Blasius assumed the free stream velocity U_∞ to be constant outside the boundary layer and not a function of x; consequently, dU_∞/dx equals zero. The revised integral momentum equation is:

$$d\delta_2/dx = \tau_o/\rho U_\infty^2 \tag{1}$$

In order to solve this equation using the approximate method, a velocity profile must be assumed which is shown in Equation 25 from Chapter 3. After applying the conditions stated above, the velocity profile equation becomes:

$$u/U_\infty = (24\eta + 6\eta^4)/18 \tag{2}$$

The boundary conditions for this flow are:

$y = 0\ (\eta = 0), \quad u = 0\ (f(\eta) = 0)$

$y = \delta\ (\eta = 1), \quad u = U_\infty\ (f(\eta) = 1)$

where $u/U_\infty = f(\eta) = (24\eta + 6\eta^4)/18$

By definition, shear stress at the surface of the plate τ_o is:

$$\tau_o = -\mu\left(\frac{\partial u}{\partial y}\right)_o \tag{3}$$

Using the chain rule and the expression, $\eta = y/\delta$, Equation 3 is revised as:

$$\tau_o = -\mu\left(\frac{\partial u}{\partial \eta}\right)_o\left(\frac{\partial \eta}{\partial y}\right)_o$$

Expanding τ_o with the use of the expression, $u = U_\infty f(\eta)$, yields:

$$\tau_o = -\mu\left(\frac{\partial U_\infty f(\eta)}{\partial \eta}\right)_o \left(\frac{\partial \eta}{\partial y}\right)_o \tag{4}$$

Substituting Equation 2 into Equation 4 gives the shear stress at the surface with no suction and no pressure gradient applied. Equation 5 is used later to derive the coefficient of drag.

$$\tau_o = -\mu U_\infty (24/18)(1/\delta) \tag{5}$$

The objective now is to find the value of δ in Equation 5 so it can be substituted into the shear stress equation.

Taking the momentum thickness δ_2 as specified in Equation 35 from Chapter 3 and applying to it the conditions of no suction and no pressure gradient, simplifies the momentum thickness δ_2/δ as:

$$\delta_2/\delta = 0.143 \tag{6}$$

Substituting Equations 5 and 6 into Equation 1 gives:

$$\frac{d}{dx}(0.143\delta) = \frac{1}{\rho U_\infty^{\;2}}\left(-\mu U_\infty \frac{24}{18}\frac{1}{\delta}\right) \tag{7}$$

Equation 7 is then rearranged to:

$$U_\infty^{\;2}\frac{d}{dx}(0.143\delta) = \frac{24}{18}\frac{\nu U_\infty}{\delta} \tag{8}$$

Simplifying and integrating Equation 8 yields:

$$0.143\frac{\delta^2}{2} = \frac{24}{18}\frac{\nu x}{U_\infty} \tag{9}$$

Equation 9 finally yields the thesis boundary layer thickness δ that can be compared with the boundary layer thickness derived by Blasius. The term δ is a function of x, ($\delta = \delta(x)$). From Equation 9, $\delta(x)$ is found as:

$$\delta(x) = 4.318 \sqrt{\nu x / U_\infty} \tag{10}$$

Blasius' solution for boundary layer thickness is:

$$\delta(x) \approx 5.0 \sqrt{\nu x / U_\infty} \tag{11}$$

As observed when comparing Equations 10 and 11, the Blasius solution and this thesis solution for boundary layer thickness differ by thirteen percent. The non-dimensional velocity profile u / U_∞ can be solved by substituting Equation 10 into Equation 2 where $\eta = y / \delta(x)$. The velocity profile then becomes:

$$\frac{u}{U_\infty} = \frac{24}{18}\left(\frac{y}{4.318}\sqrt{\frac{U_\infty}{\nu x}}\right) + \frac{6}{18}\left(\frac{y}{4.318}\sqrt{\frac{U_\infty}{\nu x}}\right)^4$$

where the limits are:

$$0 \leq y \sqrt{U_\infty / \nu x} \leq 5$$

Figure 4.1 shows how the approximate velocity profile compares with Blasius. The comparison is close for a no suction and no pressure gradient flow.

The shear stress at the wall for no suction and no pressure gradient is found by substituting Equation 10 into Equation 5 which yields:

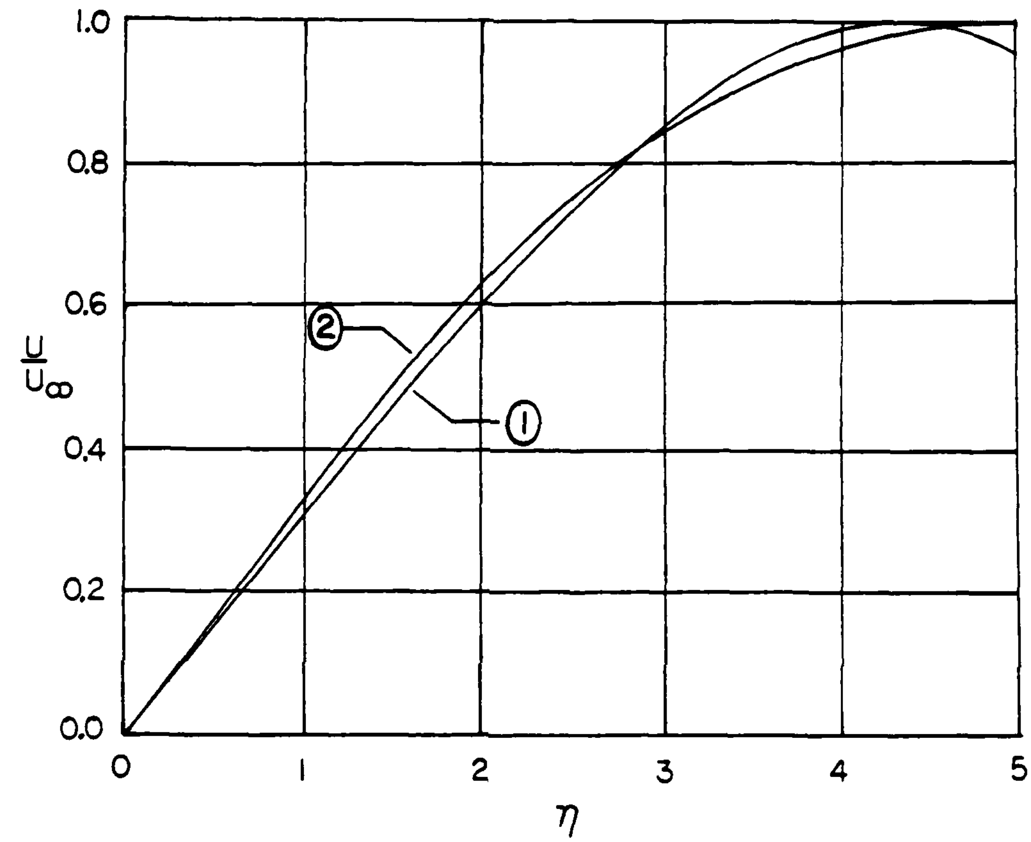

Figure 4.1

Comparison of Thesis and Blasius Velocity Solutions

No Suction, No Pressure Gradient

1--Thesis

2--Blasius

$$\tau_0 = -\mu U_\infty \left(\frac{24}{18}\right)\left(\frac{1}{4.318} \quad \frac{1}{\sqrt{x\nu / U_\infty}}\right)$$

The above equation then reduces to:

$$\tau_0 = -0.309\ \mu U_\infty \sqrt{U_\infty / x\nu} \qquad (12)$$

Equation 12 is the solution of shear stress by the approximate method with no suction and no pressure gradient. τ_o has a 6.9 percent error when compared to Blasius.

The displacement thickness δ_1 is solved in Equation 32 from Chapter 3. Since δ_1 is to be compared with Blasius, again the terms M and N are assumed zero. The result is $\delta_1 = (0.4)\delta$. The term δ is equal to $4.318 \sqrt{\nu x / U_\infty}$. Substituting δ into the equation for δ_1 gives:

$$\delta_1 = 1.727 \sqrt{\nu X / U_\infty} \qquad (13)$$

The values of δ_1 from Blasius' solution and Equation 13 are almost exact. The same procedures used to find δ_1, τ_o, and u / U_∞ are used to find the momentum thickness δ_2. Taking Equation 35 from Chapter 3 and assuming no suction and no pressure gradient then $\delta_2 = (0.143)(\delta)$. Therefore δ_2 becomes:

$$\delta_2 = 0.617 \sqrt{\nu X / U_\infty} \qquad (14)$$

The difference between Equation 14 and Blasius' solution is 6.8 percent.

Other approximations have been made and compared with Blasius, but none of the other solutions included suction and pressure gradient simultaneously. Table 4.2 shows how

other approximations made by Pohlhausen and Prandtl compared with Blasius and the proposed approximation of this thesis.[10] Figure 4.2 is a graphical display of Table 4.1. Figure 4.1 compares Blasius' solution and the solution of this thesis. The thesis solution is quite similar when it is reduced to a no suction and no pressure gradient condition.

	Blasius	Thesis	Linear	Prandtl	Pohlhausen
$\delta(x)$	5.0	4.318	3.464	4.64	5.835
δ_1	1.729	1.727	1.732	1.740	1.7505
δ_2	0.664	0.617	0.577	0.645	0.687
τ_o	0.332	0.309	0.288	0.322	0.343
$f'(0)$	--	1.333	1.0	1.5	2.0

Table 4.1

Comparison of the Integral Momentum Method and the Blasius Solution

Key for table symbols:

$$\delta(x) = K_1 \sqrt{\nu x / U_\infty}$$

$$\delta_1 = K_2 \sqrt{\nu x / U_\infty}$$

$$\delta_2 = K_3 \sqrt{\nu x / U_\infty}$$

$$\tau_o = K_4 (u U_\infty) \sqrt{U_\infty / (x \nu)}$$

$$f'(0) = \partial(u / U_\infty) / \partial\eta$$

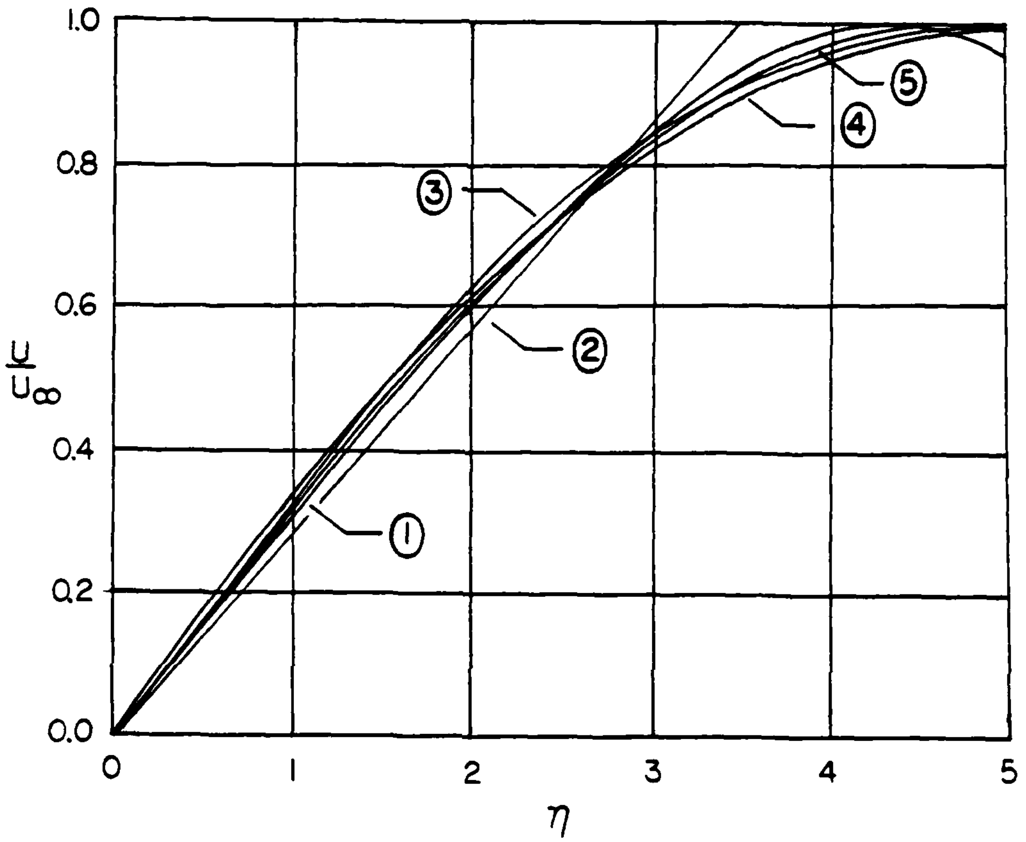

Figure 4.2

Comparison of Velocity Approximate Methods

with the Blasius Solution

No Suction, No Pressure Gradient

1--Thesis

2--Linear Approximation

3--Blasius

4--Pohlhausen

5--Prandtl

The coefficient of drag C_D is a function of shear stress and the area of the plate yielding:[11]

$$C_D = \frac{\tau_0 (AREA)}{(1/2)\rho U_\infty^2 (AREA)} = \frac{2\tau_0}{\rho U_\infty^2} \qquad (15)$$

Since only one side of the plate is being analyzed, it would be appropriate to use $(1/2)C_D$ so that $C_D = \tau_o / \rho U_\infty^2$. Taking Equation 5 and rearranging it to solve for τ_o, where suction and pressure gradient are present, τ_o becomes:

$$\tau_0 = \frac{\rho U_\infty^2}{\sqrt{Re}} \left(\frac{24 + 6N + MN}{18 + 6M + M^2} \right) \qquad (16)$$

The term, Re, is Reynolds number and it is defined as $Re = U_\infty x / \nu$.

Substituting one-half of Equation 16 into Equation 15 gives:

$$C_D = \left(\frac{24 + 6N + MN}{18 + 6M + M^2} \right) \frac{1}{\sqrt{Re}} \qquad (17)$$

To compare Equation 17 with the Blasius solution the terms N and M are equated to zero. Figures 4.3 and 4.4 show the Blasius and thesis solutions to be equal at M and N equal to zero. Equation 17 will deviate from the Blasius solution as the term M is varied. This is shown in Figure 4.3. As suction is increased along with Re, the coefficient of drag is limited by the term M. Figure 4.5 shows that when M is approximately equal to -3 the C_D plot reaches its maximum value.

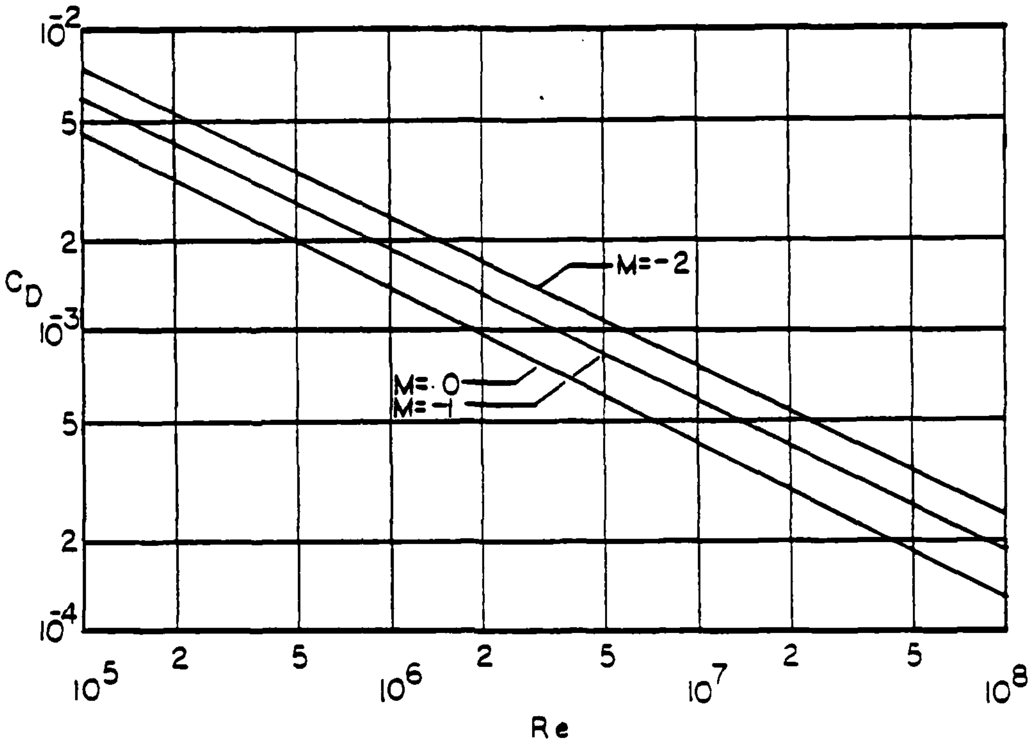

Figure 4.3

Coefficient of Drag Versus Reynolds Number with Suction

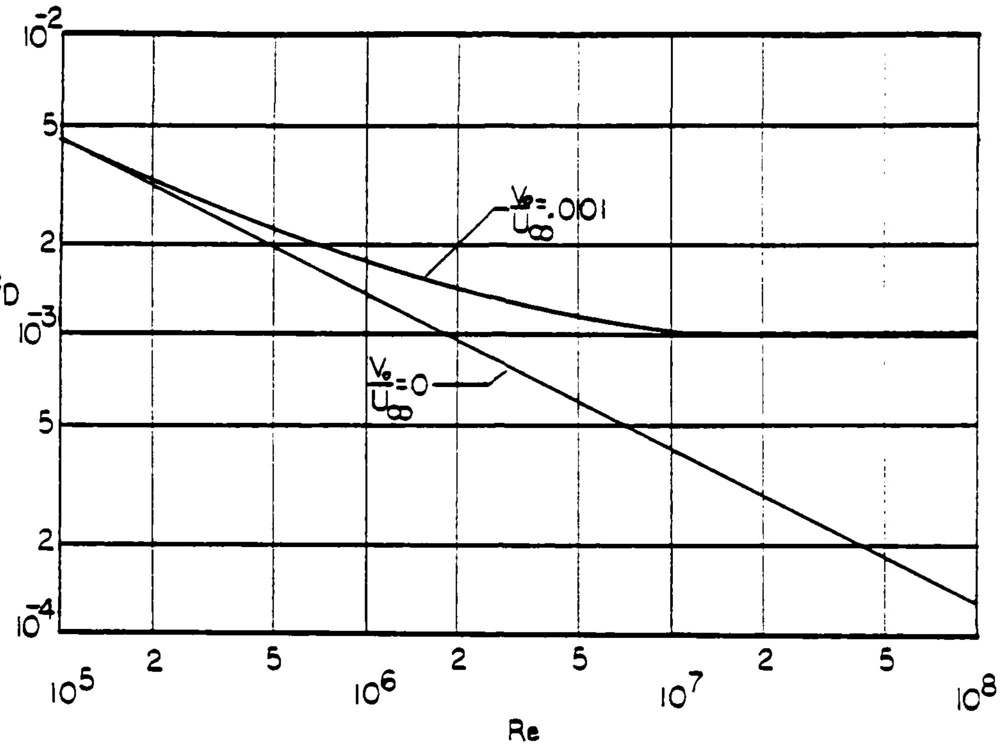

Figure 4.4

Coefficient of Drag Versus Reynolds

Number with Suction

Source: J. Kestin, trans., Boundary-Layer Theory, by Hermann Schlichting , 6th rev. ed. (New York: McGraw Hill, 1968), p. 371.

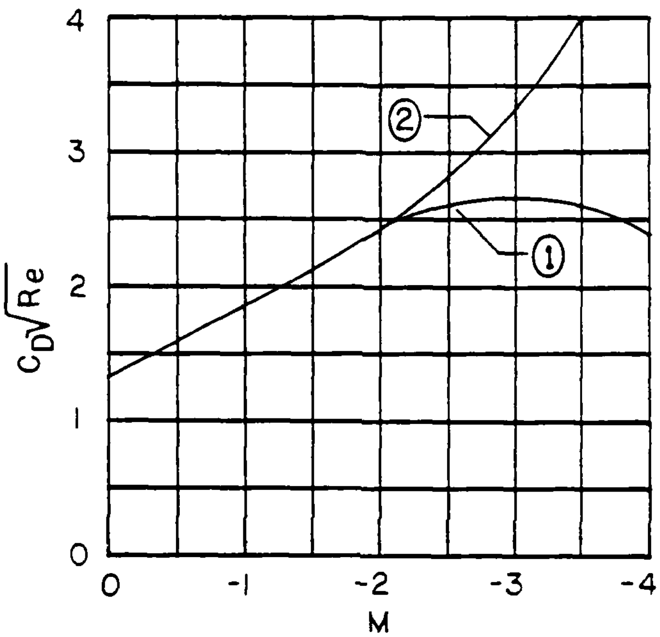

Figure 4.5

Coefficient of Drag Versus Suction

Parameter for N = 0

1--Thesis

2--Edwards

As negative M further increases the calculated drag coefficient decreases. For these reasons the solution developed in this thesis is not valid beyond M = -2.[12] Figure 4.5 shows how the coefficient of drag is affected by the suction term M when pressure gradient is not present. This thesis agrees exactly with the results presented by J.B. Edwards in an article on laminar flow.[13] The coefficient of drag between the two studies begins diverging at M = -2; therefore the value of M in this thesis is limited to a value of -2 and greater. Figures 4.3 and 4.4 also confirm this fact. When $v_o/U_\infty = -0.001$ at $Re = 10^6$, which is equivalent to M = -1, the two plots agree. When $Re = 10^5$ and $v_o/U_\infty = -0.001$, M becomes -0.316. This comparison is quite reasonable between the graphs only up to Reynolds number equal to 5×10^6 and for M equivalent to or greater than -2.

Suction varies with Reynolds number in order to maintain a constant suction parameter M. It is observed that too much suction can be disruptive to a laminar flow. An optimum suction to keep a flow laminar for $Re = 10^8$ is found to be $v_o/U_\infty = 1.3 \times 10^{-4}$. In summary only a small amount of suction is necessary to keep a flow laminar that is normally considered in the non-laminar range ($Re > 2.5 \times 10^6$) The necessary v_o/U_∞ that will keep a flow laminar is effective as long as it does not exceed M = -2.

The solution deviates at values of $M < -2$ and does not reflect a true reaction beyond the limit. Edwards' equation was also limited at $M = -2$, but he was able to overcome that limitation with another function.

4.2 Suction and Pressure Gradient at Separation

From Chapter 3 the velocity profile was in the form of:

$$u(\eta) = U_\infty(a\eta + b\eta^2 + c\eta^3 + d\eta^4) \tag{18}$$

On the right hand side of Equation 18 each coefficient has the term, $D = 18 + 6M + M^2$, in the denominator which is shown in Equation 25 from Chapter 3. The denominator has no real roots; the imaginary roots are $3 \pm 3\sqrt{-1}$.[14] Consequently Equation 25 will not be infinite for any value of M where M is a function of suction v_o. Torda observed that the velocity profile would approach infinity if certain boundary conditions were used. This also caused the coefficient a to be independent of suction which meant that the velocity and separation would not be affected by suction. Torda later suggested the new boundary conditions used in Chapter 3. The suggested boundary conditions did correct the problem of coefficient a approaching infinity caused by M in the denominator.[15]

The separation point is located where the wall shear stress is zero. Shear stress is $\mu(\partial u/\partial\eta)(\partial\eta/\partial y)$.

Taking Equation 25 from Chapter 3 and substituting it into the expression, $\partial u / \partial \eta = 0$, yields:

$$\left.\frac{\partial u}{\partial \eta}\right|_{o} = 0 = \frac{24 + 6N + MN}{(18 + 6M + M^2)\delta} \tag{19}$$

Expanding Equation 19 and solving for N gives:

$$N = -24 / (6 + M) \tag{20}$$

Equation 20 describes the pressure gradient as a function of suction where $M = v_o \delta / \nu$. Pohlhausen's solution to a similar flow without suction was $N = -12$. Torda also initially derived the separation pressure gradient as $N = -12$, but the boundary conditions caused the suction to be ineffective on the flow and separation. Appendix C shows the difference between the solutions of Pohlhausen and this thesis for separation point.

Table 4.2 lists the pressure gradient at separation for each suction term M. The term M can also apply to blowing. Blowing is positive M and suction is negative M.

M	-2	-1	0	1	2
N	-6	-4.8	-4	-3.42	-2.66

Table 4.2

Separation Pressure Gradient with Suction

To have separation, adverse pressure gradient must

exist. As suction increases the fluid flow is able to remain laminar with more adverse pressure which means that the separation of flow can be averted or delayed.

Figure 4.6 depicts Pohlhausen's velocity profile. Figures 4.7 through 4.9 show the velocity profile of this thesis as a function of the suction term M and various values of pressure gradient until separation occurs.

4.3 Separation Velocity

As a fluid flows over a flat plate, drag will eventually cause the pressure gradient to become adverse and this will cause separation to occur. To help prevent or delay separation, suction is used. It is possible to find the amount of suction velocity required to keep the flow laminar and free from separation. Prandtl was able to find the velocity of suction that would prevent separation by using the momentum equation solved in Equation 11, Chapter 2.[16] Prandtl used Pohlhausen's velocity profile which is a function of pressure gradient only. In this thesis to find the effects of suction velocity and pressure gradient, Equation 25 from Chapter 3 is used for the velocity profile. Table 4.2 shows that with the suction parameter being zero, the critical pressure gradient at separation is N = -4. Substituting M = 0 and N = -4 into the velocity profile equation the revised equation for velocity is:

$$u / U_\infty = 2\eta^2 - \eta^4 \qquad (21)$$

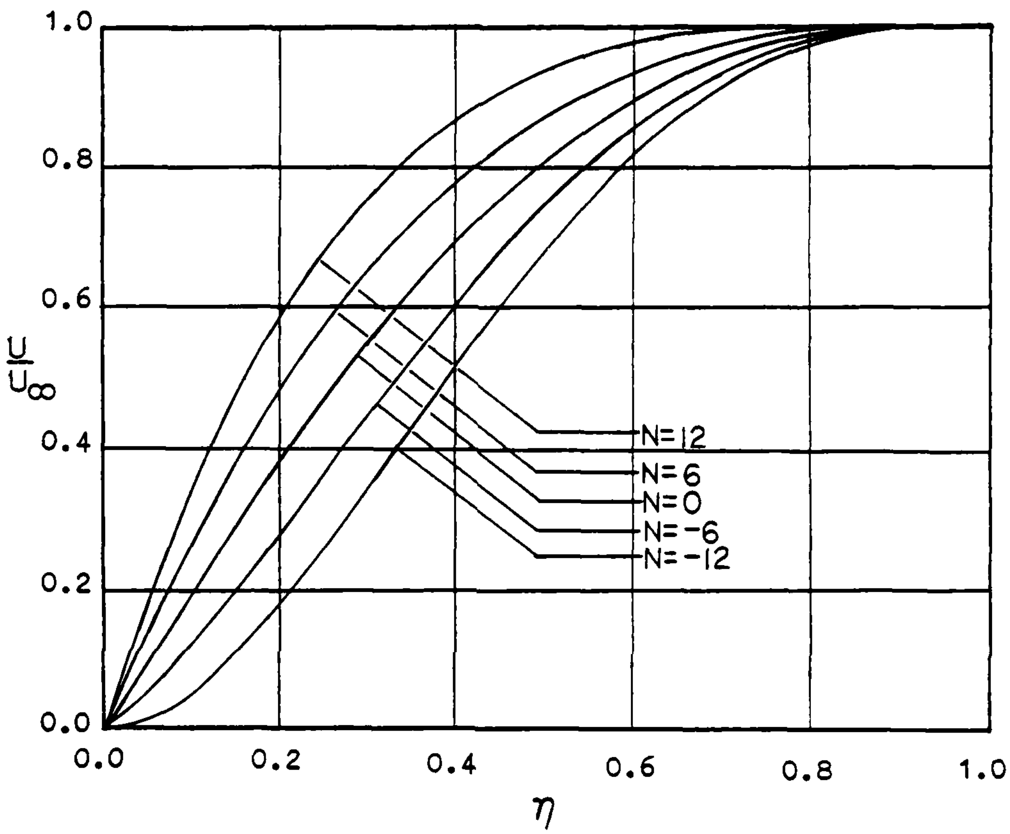

Figure 4.6

Effect of Pressure Gradient on

Velocity Distribution

(Pohlhausen's Solution)

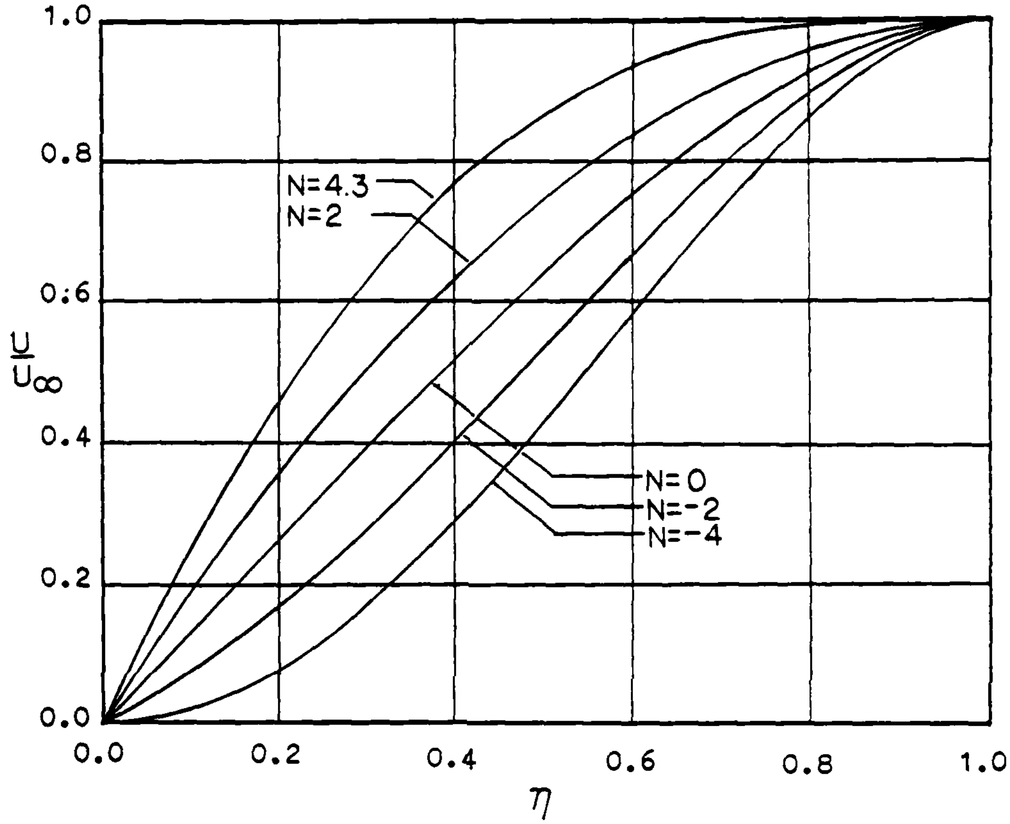

Figure 4.7

Effect of Pressure Gradient on

Velocity Distribution

M = 0

Figure note: $(u/U_\infty) > 1$, which is unsteady flow, occurs when N > 4.3.

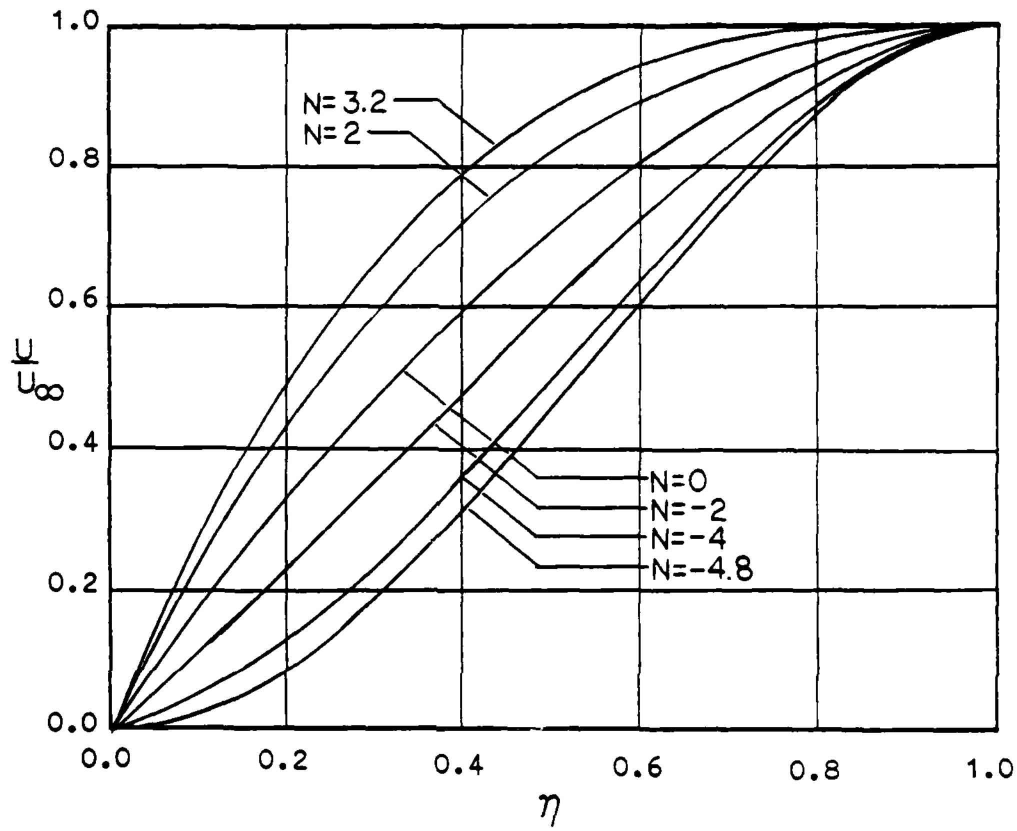

Figure 4.8

Effect of Pressure Gradient on

Velocity Distribution

M = -1

Figure note: $(u/U_\infty) > 1$, which is unsteady flow, occurs when N > 3.2.

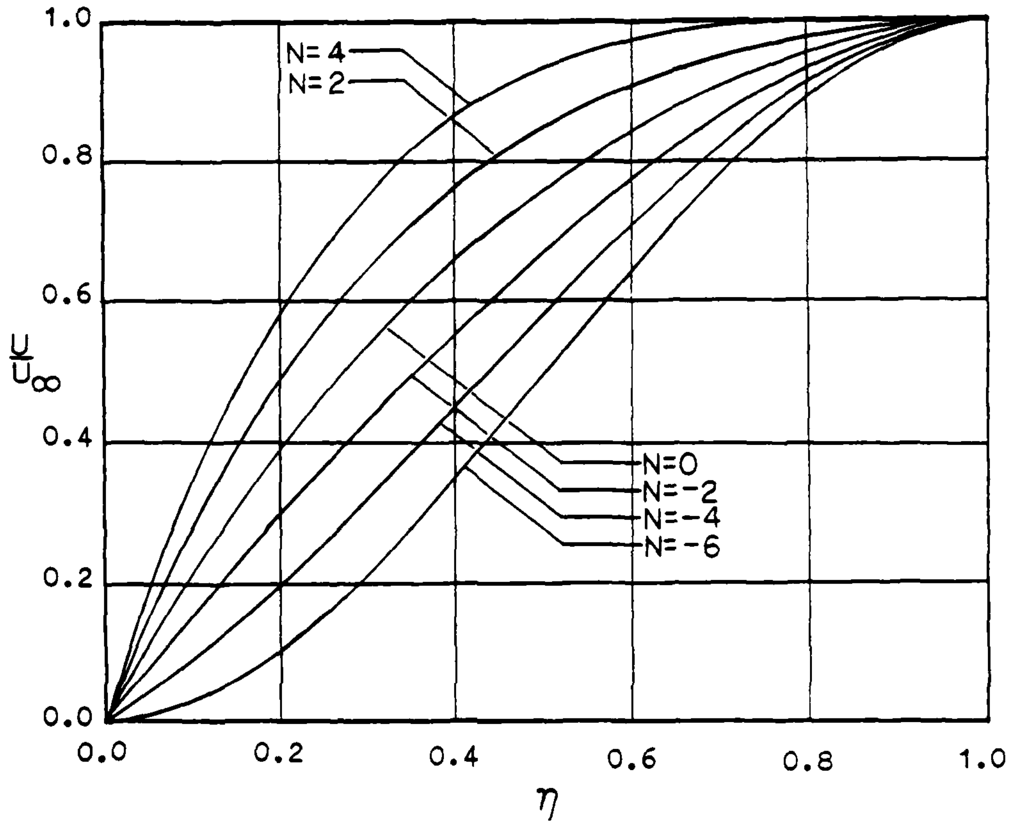

Figure 4.9

Effects of Pressure Gradient on

Velocity Distribution

M = -2

Figure note: $(u/U_\infty) > 1$, which is unsteady flow, results when N > 4.8.

Equation 21 is used in the definition of displacement thickness δ_1 giving:

$$\frac{\delta_1}{\delta} = \int_0^1 \left(1 - \frac{u}{U_\infty}\right) d\eta = \int_0^1 (1 - 2\eta^2 + \eta^4) d\eta \qquad (22)$$

Solving Equation 22 yields $\delta_1 / \delta = 0.5333$. To verify the above calculations, Equation 29 from Chapter 3 shows the same results. To find the momentum thickness δ_2, Equation 22 is used in the definition of δ_2 giving:

$$\frac{\delta_2}{\delta} = \int_0^1 \left[\frac{u}{U_\infty} - \left(\frac{u}{U_\infty}\right)^2\right] d\eta = \int_0^1 (2\eta^2 - \eta^4 - [4\eta^4 - 4\eta^6 + \eta^8]) d\eta \qquad (23)$$

Equation 23 yields $\delta_2 = 0.1269$. To check the thesis results, Equation 35 from Chapter 3 was compared and gave the same answer.

As stated and assumed by Prandtl the momentum boundary layer thickness is assumed constant which means that $d(\delta_2)/dx = 0$. In a general flow over an arbitrarily shaped body, the free stream velocity will vary $(U_\infty(x))$. It is known that $\tau_0 / \rho = 0$ at separation, therefore the integral momentum equation stated early in this chapter reduces to:

$$\frac{\delta_2}{U_\infty}\left(\frac{dU_\infty}{dx}\right)\left(2 + \frac{\delta_1}{\delta_2}\right) + \frac{v_0}{U_\infty} = 0$$

Solving for the suction velocity yields:

$$v_0 = -(2\delta_2 + \delta_1) dU_\infty / dx \qquad (24)$$

Substituting $\delta_1/\delta = 0.5333$ and $\delta_2/\delta = 0.1269$ into Equation 24 yields:

$$v_o = -0.7871\, \delta dU_\infty/dx \tag{25}$$

Applying the conditions just stated to the original boundary layer equation, Equation 3 from Chapter 2; and knowing that the velocity u is not a function of the plate length $x(\partial u/\partial x = 0)$; the revised momentum equation becomes:

$$v_o\left(\frac{\partial u}{\partial y}\right)_o = U_\infty\frac{dU_\infty}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_o \tag{26}$$

As stated earlier in this thesis:

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial u}{\partial \eta}\frac{1}{\delta}$$

Substituting Equation 21 into Equation 26 yields:

$$\partial u/\partial y = (U_\infty/\delta)(4\eta - 4\eta^3) \tag{27}$$

Taking the partial derivitive of Equation 27 at the plate surface gives:

$$(\partial^2 u/\partial y^2)_o = 4(U_\infty/\delta^2) \tag{28}$$

At the surface, $y = 0$, $(\partial u/\partial y)_o = 0$. Putting Equations 27 and 28 into Equation 26 and solving for δ yields:

$$\delta = 2\sqrt{-\nu(dx/dU_\infty)} \tag{29}$$

Substituting Equation 29 into Equation 25 gives the required suction velocity that will enable a flow to remain laminar at the separation point. The result is:

$$v_o = -1.574\sqrt{-\nu dU_\infty/dx} \tag{30}$$

Prandtl using Pohlhausen's separation profile showed that suction can be used to prevent separation. Howarth's separation profile which is more accurate showed at separation that:

$$v_o = -1.687 \sqrt{-\nu dU_\infty / dx} \qquad (31)$$

Howarth assumed δ_1, δ_2, and dU_∞ / dx to be constant and worked the momentum equation with zero suction.[17] The thesis Equation 30 does agree with Howarth's Equation 31 quite closely, and it is clear that this thesis solution for a non-suction flow does agree with existing data.

This thesis shows that as suction is increased separation is delayed. Therefore suction will delay the separation and keep the flow laminar until the critical stage of separation is exceeded. Table 4.3 summarizes the data computed for other various suction values. As the suction increases the boundary layer thickness increases up to the point of separation. The increase of adverse pressure gradient for increasing suction explains the increasing boundary layer thickness at separation. The suction velocity permits the flow to remain laminar longer thus making the boundary layer thickness appear to increase.

4.4 Displacement Thickness

In Chapter 3 the displacement thickness δ_1 was derived in terms of suction and pressure gradient. As described before, δ_1 is the mass flow deficit in the boundary layer.

			①	②	③
M	N	δ_1/δ	$(\delta_1 + 2\delta_2)$	$[K_1]$	$[K_2]$
0	-4.0	0.5333	0.7871	2.000	-1.5742
-1	-4.8	0.52	0.776	2.190	-1.699
-2	-6.0	0.5	0.756	2.449	-1.851

Table 4.3

Separation of Flow Data for

Various Suction Velocities

Key for table symbols:

① $v_0 = [\delta_1 + 2\delta_2]\delta dU_\infty/dx$

② $\delta = [K_1]\sqrt{-\nu dx/dU_\infty}$

③ $v_0 = [K_2]\sqrt{-\nu dU_\infty/dx}$

When suction is applied the displacement thickness should decrease since the mass flow essentially has been displaced closer to the flat plate due to the suction force. The previous section showed how the velocity flow compared to Blasius when the suction and pressure gradient were zeroed out. Equation 29 from Chapter 3 gives the displacement thickness as:

$$\delta_1 = (432 - 36N - 9MN + 192M + 36M^2)/60D$$

When M and N each equal zero, $\delta_1 = (0.4)\delta_1$. Substituting $\delta = 4.318\sqrt{\nu x/U_\infty}$ into the δ_1 equation yields: $\delta_1 = 1.727\sqrt{\nu x/U_\infty}$. This solution is in excellent

agreement with Blasius'solution for δ_1. The same procedure is used for various values of the suction parameter M and the pressure gradient parameter N.

The limits for M and N are shown in Table 4.2. It serves no purpose to go beyond the separation point; a steady flow is assumed so that u/U_∞ is not greater than one and not less than zero. Figure 4.10 graphically shows how the displacement thickness decreases as the suction velocity increases. To counteract the suction force, pressure gradient is adjusted until the point of separation is reached. Increasing adverse pressure gradient will cause the displacement thickness to increase. Figures 4.11 and 4.12 show how the pressure gradient affects the mass flow for varying suction parameters. Figures 4.10 through 4.12 are assumed to have the same boundary layer thickness as Blasius so that the effects of suction and pressure gradient on a flow can be compared.

4.5 Momentum Thickness

The separaton profile of a flow can also be analyzed through momentum flux which is explained by momentum thickness. The flow of a fluid near the surface where the suction is applied may have insufficient momentum to continue. The result may be either a stalled flow or a turbulent flow.

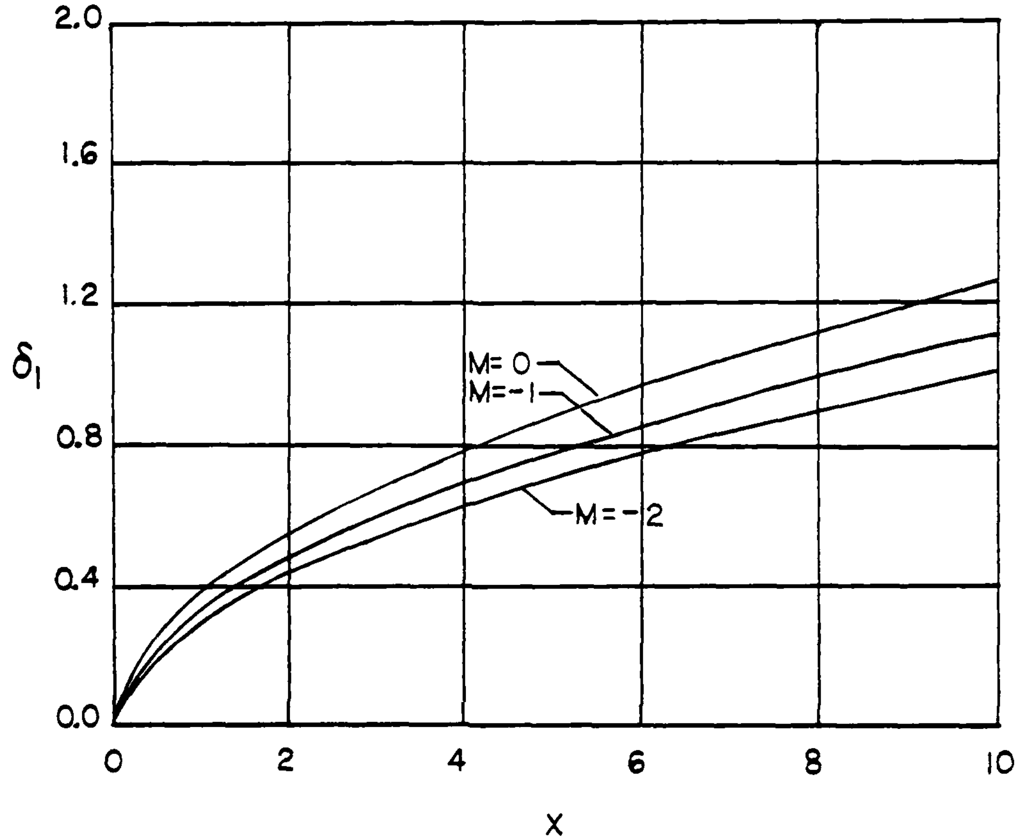

Figure 4.10

Displacement Thickness Versus Length X (inches)

for Different Suction Values

No Pressure Gradient

N = 0

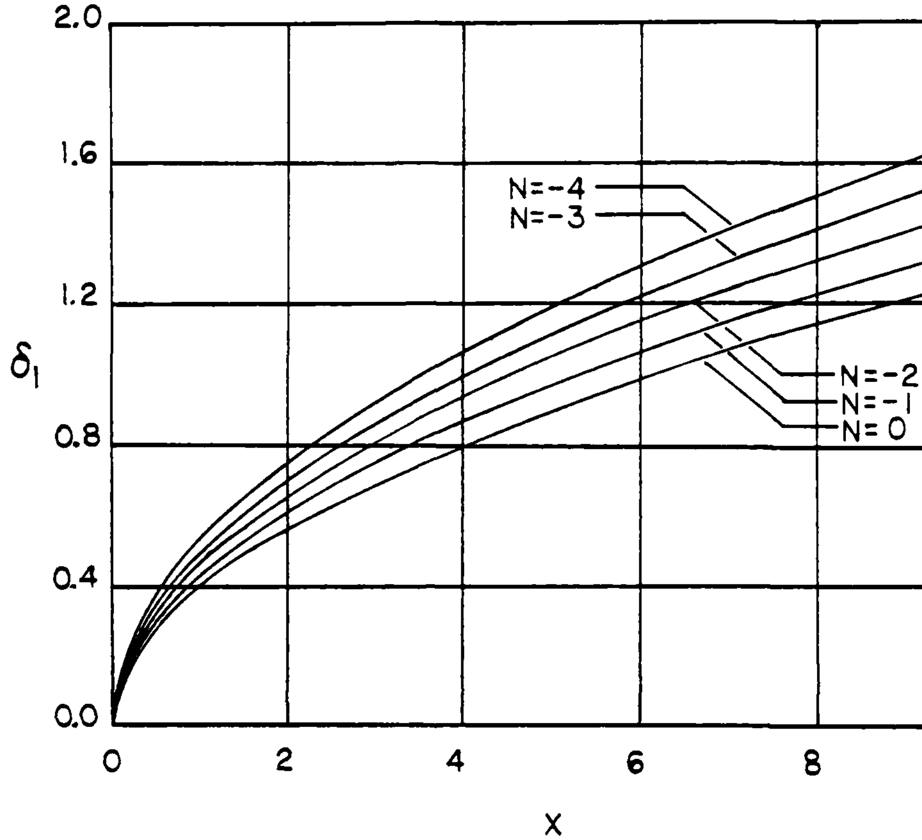

Figure 4.11

Displacement Thickness Versus Length X (inches)

for Different Pressure Gradients

No Suction

M = 8

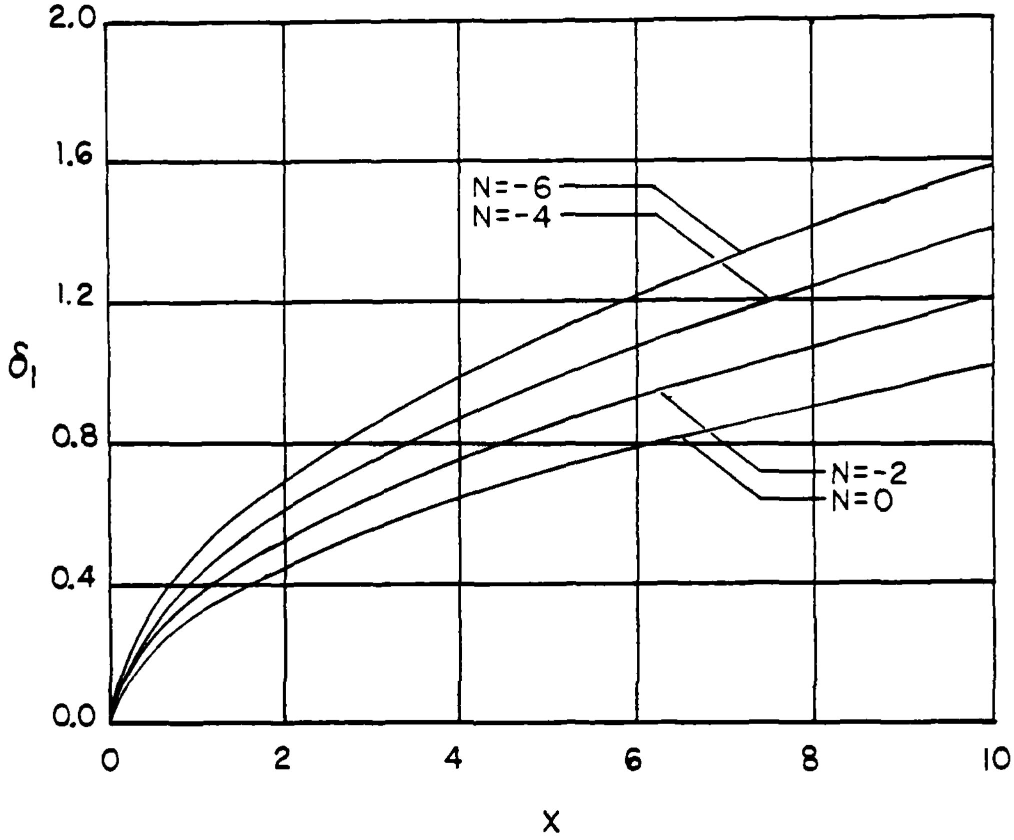

Figure 4.12

Displacement Thickness Versus Length X (inches)

for Different Pressure Gradients

M = -2

If the Reynolds number is high enough to keep the flow moving, which is normally the case, the flow becomes turbulent.

Equation 35 from Chapter 3 is the thesis solution for momentum thickness. It was shown how closely this solution compared with the Blasius solution when suction and pressure gradient were zero. Comparing the thesis solution with Pohlhausen's solution showed the solutions to react similarly when adverse pressure was applied.

Figures 4.13 through 4.15 show the reactions as suction and pressure gradient change. Increasing pressure gradient increases the momentum thickness initially, however the thickness decreases at the half-way mark toward separation pressure gradient. The half-way mark is where the flow profile changes shape and direction. The momentum thicknesses at zero pressure gradient and at separation pressure gradient approach each other as suction velocity increases. This indicates that extreme suction ($M < -2$) can cause a flow to transition into a turbulent flow. How suction and Reynolds number affect each other is explained in Section 4.1.

Figures 4.3 and 4.5 show that increasing suction increases the skin friction which also relates to the causes of separation.

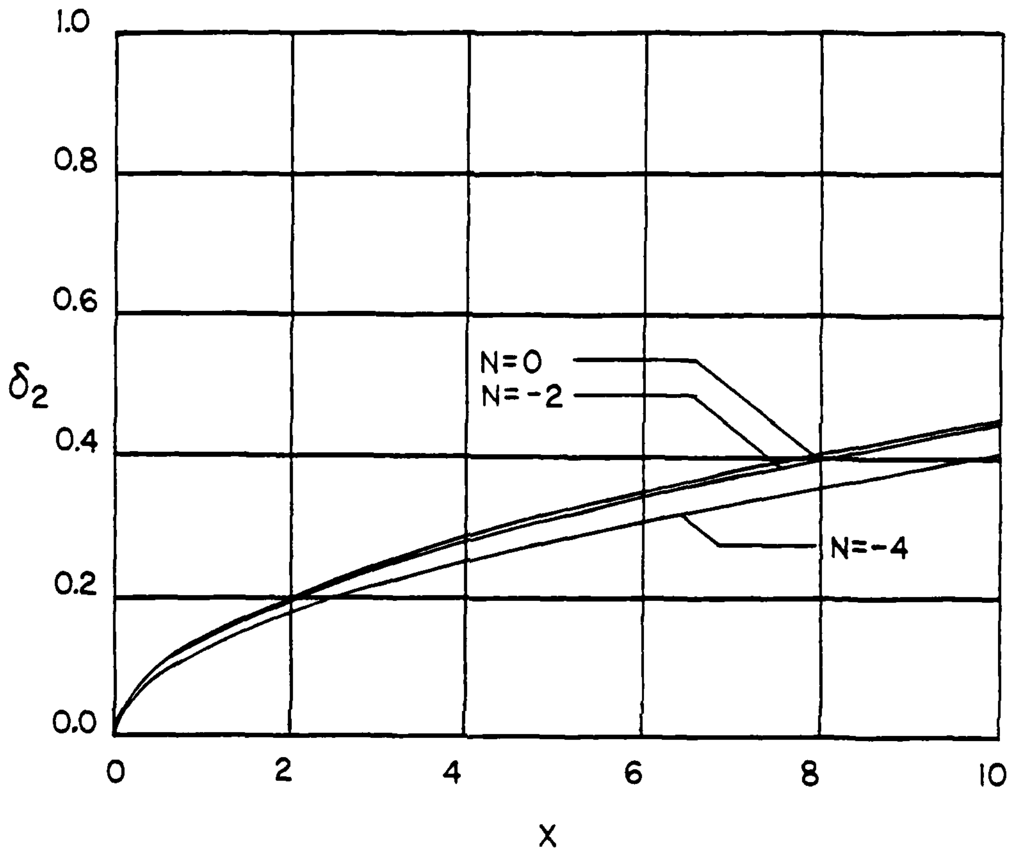

Figure 4.13

Momentum Thickness Versus Length X (inches)

M = 8

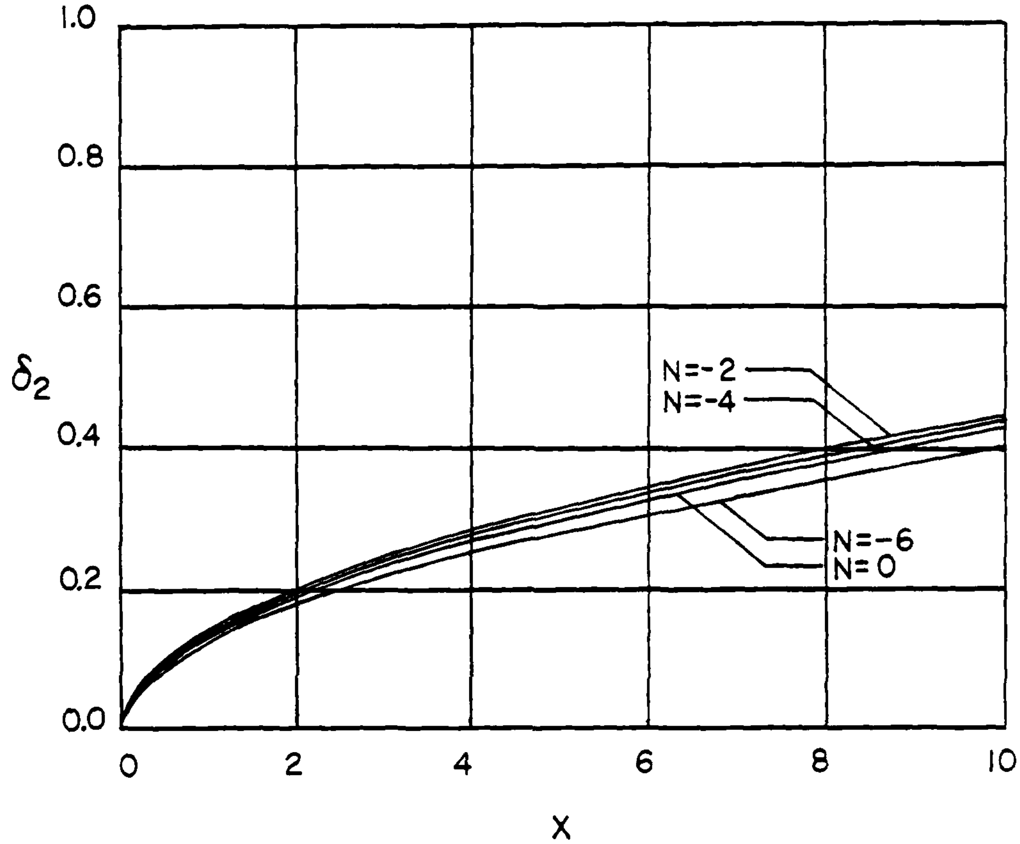

Figure 4.14

Momentum Thickness Versus Length X (inches)

M = -2

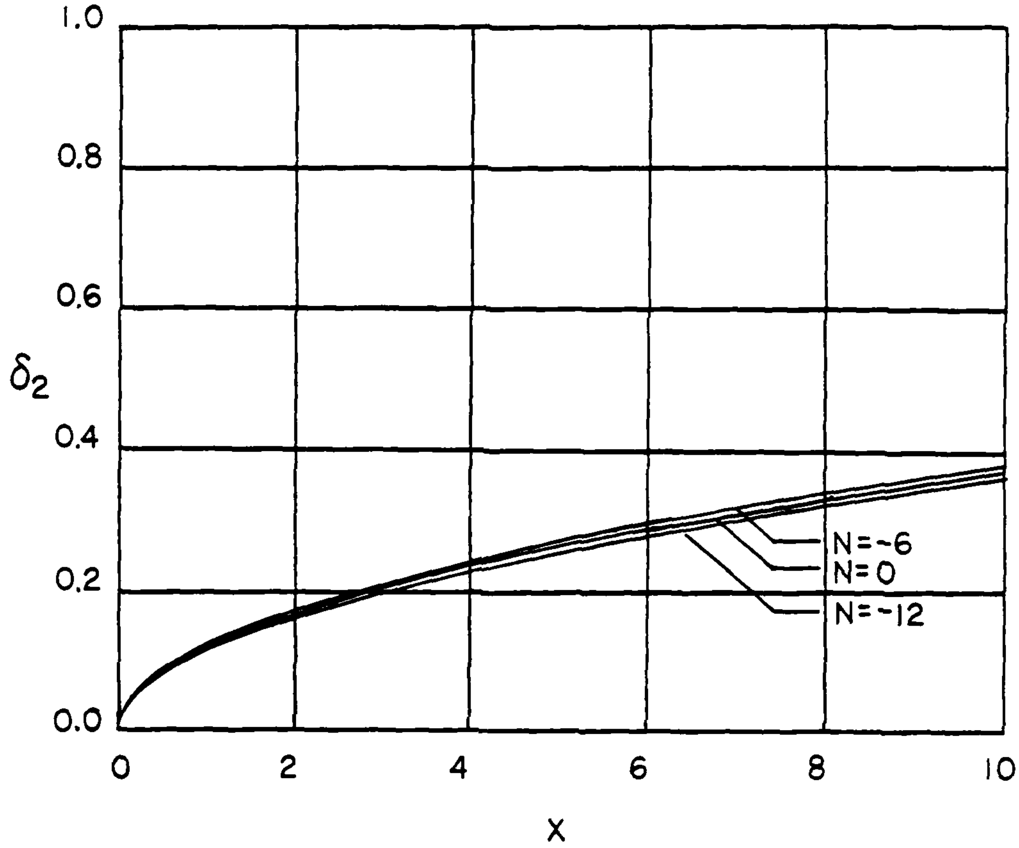

Figure 4.15

Momentum Thickness Versus Length X (inches)

(Pohlhausen Solution)

The values of v_o/U_∞ and Reynolds number dictate the value of M and whether or not the solution is in the laminar region. Any value of M less than -2 is considered in the turbulent region. The thesis solution pertains only to laminar flow. Figures 4.13 through 4.15 show that momentum thickness is affected by suction flow, and that at the separation point the momentum thickness deviates very slightly.

CHAPTER 5

Conclusions

The purpose of this thesis was to study suction flow over a flat plate and to observe the flow reaction with pressure gradient. The objective was to derive an approximate velocity profile in terms of pressure gradient and suction parameters. This was accomplished by using Pohlhausen's assumed fourth order polynomial with different boundary layer conditions later proposed by Torda. The approximate velocity profile derived in Chapter 3 was used to further analyze the derived solution.

The approximate velocity profile clearly showed that suction velocity is benefical in preventing or delaying flow separation. This thesis solution only applied to small values of suction ($v_o / U_\infty = 0.002$ or less) and to $Re = 10^8$ or less. Figures 4.7 through 4.9 amplified the point of how suction and pressure gradient can manipulate a flow reaction up to the separation point. In observing the graphs for drag caused by friction the thesis was less accurate for the value of M less than -2 which is in the realm of turbulent flow. The thesis did not address turbulence nor was it applicable for turbulent flow.

As the suction parameter decreased (more suction added) the coefficient of drag increased. The combined reactions of suction and friction helped to delay flow separation. Suction can force the flow to separate simply because the momentum of the flow exceeds the viscous force. It was observed that the laminar range for Reynolds number extended as suction velocity was added. Normally the limit for zero suction, laminar flow is Re = 2.5 X 10^6. Adding suction where v_o / U_∞ = 0.001 would keep the flow laminar for Reynolds number up to 5 X 10^6. As the Reynolds number increased to 10^8, less suction, v_o / U_∞ = 1.3 X 10^{-4} , was needed to keep the flow laminar. Adding more suction at this Reynolds number would cause a non-laminar situation. This shows that suction does help keep a flow laminar for increasing Reynolds number but with limitations. This set the limit of the approximate solution at M = -2. The figures showing velocity profile clearly showed that at M = -2 the suction flow patterns changed direction which also indicates the same limit to this approximate solution.

The solutions behaved with the predicted patterns for suction flow and changing pressure gradient within the set limits. For comparing the thesis solution with other proven solutions a common denominator of zero suction was studied. Pohlhausen showed separation to occur at N = -12, the thesis at N = -4. The difference is caused by the use of different

boundary conditions. When Pohlhausen's conditions were used the solutions agreed at N = -12, however the coefficient used to find separation values eventually equaled infinity. Consequently different boundary conditions were used to overcome this as proposed by Torda. The thesis agreed with Howarth's prediction for the necessary suction velocity needed to maintain laminar flow at separation (N = -4). As suction was added the separation was moved further downstream by simply permitting more adverse pressure gradient to react on the flow.

The reason for using a fourth order polynomial to approximate the velocity profile was that it would yield a zero second-derivative at the plate surface which was a boundary condition. The approximate solution implies that a definite boundary layer exists; whereas an exact solution assumes that the boundary layer extends indefinitey.

The thesis solution has an advantage in that a boundary layer can be assumed and the necessary suction and pressure gradient parameters can be substituted into the equation to analyze the flow and thickness. Another advantage of the thesis is that the assumed boundary conditions prevented the velocity profile from going infinite for any suction parameter. The boundary conditions proposed by Pohlhausen caused the velocity flow to become infinite for M = -6 which is not a realistic solution, thereby justifying a new set of

boundary conditions.

A problem encountered was the definitions of terms from other studies. A noted difference existed in the use of the terms η and M. In this thesis $\eta = y/\delta$; in other studies $\eta = y/(2\delta)$, $\eta = y/(\sqrt{2}\,\delta)$, or $\eta = 2y/\delta$. Another difference was with the value for M; the thesis value was $M = (v_o/U)\sqrt{Re}$, whereas in other studies $M = (v_o/U)\sqrt{Re}/2$. Another noted problem was that some exact studies had their graphs and charts based on parameters that were most convenient to their studies, and it was not possible to duplicate the same parameters due to the complexity of terms in the approximate solutions. It was more convenient to compare the thesis approximate solution with other approximate solutions due to the similarity of their methods.

The following conclusions are made about the thesis approximate solution for suction flow:

1. The approximate solution is in good agreement with Blasius solution when suction and pressure gradient are zero.

2. At separation the approximate solution agreed with Howarth's prediction for a no suction flow.

3. Increasing suction velocity does help prevent or delay flow separation. Suction enables the flow to withstand higher adverse pressure further downstream where the

separation flow pattern is formed. More suction permits more adverse pressure gradient to react on the flow.

4. The suction velocity required to keep a flow laminar at separation point increased with increasing suction velocity. The cause for this is that as suction increases the separation adverse pressure gradient increases.

5. Increasing suction parameter made the velocity profile fuller but not as full as Pohlhausen's and other solutions. The velocity profile reached its maximum fullness at M = -2 before reversing the reaction of suction, thereby making the approximation solution limited.

6. Momentum flux shown by the momentum thickness varied with adverse pressure by initially increasing and then decreasing as separation flow was reached. Thesis approximate solution agreed with Pohlhausen's solution. Increasing suction parameter decreases momentum thickness.

7. A possible solution to overcome the cancellation of a term in the assumed velocity profile equation at M = 8 may be obtained by assuming a fifth order polynomial instead of a fourth order polynomial. This may also permit the assumed profile to be more full.

APPENDIX A

Flow Distribution with Pressure Gradient

Pohlhausen's approximate method was used to solve the velocity distribution and was based on von Karman's momentum equation. A fourth order polynomial was assumed by Pohlhausen to represent the flow; he worked the equation in non-dimensional form. Pohlhausen's velocity profile is:

$u/U_\infty = a\eta + b\eta^2 + c\eta^3 + d\eta^4$ where $\eta = y/\delta$

The momentum equation from Prandtl for the x-direction is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty\frac{dU_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

Boundary conditions for a steady, two-dimensional flow over a wall are:

at $y = 0$ ($\eta = 0$) $u = 0$, $v = 0$,

at $y = \delta$ ($\eta = 1$), $u = U_\infty$, $\partial u/\partial y = \partial^2 u/\partial y^2 = 0$

therefore:

$$\frac{\partial u}{\partial y} = a + 2b\eta + 3c\eta^2 + 4d\eta^3 \qquad (2)$$

$$\frac{\partial^2 u}{\partial y^2} = 2b + 6c\eta + 12d\eta^2 \qquad (3)$$

At $y = \delta$ which implies $\eta = 1$, Equations 2 and 3 become:

$$\frac{\partial u}{\partial y} = 0 = a + 2b + 3c + 4d$$

$$\frac{\partial^2 u}{\partial y^2} = 0 = 2b + 6c + 12d$$

At $y = 0$ ($\eta = 0$), Equation 3 reduces to $\partial^2 u/\partial y^2 = 2b$. From Equation 1, $U_\infty (dU_\infty /dx) = 1/\rho (dP/dx)$ and $dP/dx = 0$. The term, $U_\infty (dU_\infty /dx)$, equals zero. Consequently $\partial^2 u/\partial x^2 = 0$. Substituting the boundary conditions into Equations 1 and 3, knowing that $\partial^2 u/\partial y^2 = 2b$, coefficient b becomes zero. Solving Equations 2 and 3 simultaneously yields that $a = 2d$. At $y = \delta$ ($\eta = 1$), the equation for u/U_∞ becomes:

$$\frac{u}{U_\infty} = 1 = a + b + c + d$$

Since $b = 0$, the equation is revised to:

$$1 = a + c + d \tag{4}$$

The equation, $a = 2d$, now is substituted into Equation 4 giving: $1 = 3d + c$ (5)

Solving Equations 4 and 5 simultaneously yields $d = 1$; substituting coefficient d into the equation $a = 2d$, determines that $a = 2$. Substituting the coefficients a and d into Equation 4, solves $c = -2$. Therefore the coefficients are defined as:

$a = 2, \quad b = 0, \quad c = -2, \quad d = 1$

It is known that separation will occur with adverse pressure. The condition for separation is:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$$

To analyze the velocity profile in terms of pressure gradient each of the coefficients (a, b, c, d) can be written in terms of pressure gradient p. Redefining the coefficients to a′, b′, c′, and d′ where a′ = a + p, b′ = b + p, c′ = c + p, and d′ = d + p; will enable the velocity profile equation to be solved in terms of the dimensionless coefficient p. The coefficient p is later solved in terms of the pressure gradient N where $N = (\delta^2/\nu)dU_\infty/dx$. N multiplied by a constant is equal to p. Substituting a′ = a + p into Equation 2 at y = 0 (η = 0) will initiate the steps to solving the coeficients (a, b, c, d) in terms of pressure gradient. Due to separation at the surface, a′ = 0. Since a = 2, then p = −2. At y = δ, Equation 2 becomes:

$$0 = 2b' + 3c' + 4d' \tag{6}$$

and Equation 3 becomes:

$$0 = 2b' + 6c' + 12d' \tag{7}$$

Solving Equations 6 and 7 simultaneously, gives:

$$8d' + 3c' = 0 \tag{8}$$

At y = δ (η = 1):

$$\frac{u}{U_\infty} = 1 = b' + c' + d' \tag{9}$$

Solving Equations 7 and 9 together leaves:

$$-2 = 4c' + 10d' \quad (10)$$

When Equations 8 and 10 are solved simultaneously, $d' = 3$.

Since $d' = d + p$ and $d = 1$, then $p = 2$.

Using Equation 10 and the term $d' = 3$, then:

$$c' = -8$$

Since $c' = c + p$, which is rewritten as $-8 = -2 + p$, then:

$$p = -6$$

From Equation 9, $c' = -8$, and $d' = 3$; therefore:

$$b' = 6$$

Since $b' = b + p$ and $b = 0$, then:

$$p = 6$$

At a certain adverse pressure gradient, flow will separate. To find the pressure gradient, Equation 2 is set to equal zero. This occurs at the point when shear stress, which is proportional to $\partial u / \partial y$, equals zero.

Equation 2 at $y = 0$ ($\eta = 0$) yields:

$$\frac{\partial u}{\partial y} = 0 = a'$$

Since $p = -2$ and $a' = 0 = a + p$, then:

$$0 = 2 + (-2)$$

By letting N equal the adverse pressure gradient, separation can be found.

$$a = 2 + \frac{N}{6}$$

Therefore, $N = -12$ is the amount of adverse pressure that will start separation in a flow. The other N-dependent terms that are associated with coefficients b, c, and d are:

$$b = -\frac{N}{2}; \quad c = -2 + \frac{N}{2}; \quad d = 1 - \frac{N}{6}$$

Substituting the above terms for coefficents a, b, c, and d into the equation u/U_∞ derives Pohlhausen's approximate solution as:

$$\frac{u}{U_\infty} = 2\eta - 2\eta^3 + \eta^4 + \frac{N}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) \qquad (11)$$

By definition the equation, $\delta_1 = \int_0^\infty (1 - u/U_\infty)dy$, is the displacement thickness. Since $\eta = y/\delta$, then $dy = \delta d\eta$. Also, $u = U_\infty$ when $y = \delta$ ($\eta = 1$). The displacement thickness is rewritten as:

$$\frac{\delta_1}{\delta} = \int_0^1 \left(1 - \frac{u}{U_\infty}\right)d\eta \qquad (12)$$

Substituting and integrating Equation 11 into Equation 12 gives:

$$\frac{\delta_1}{\delta} = \left[\eta - \eta^2 + \frac{\eta^4}{2} - \frac{\eta^5}{5} - N\left(\frac{\eta^2}{2} - \eta^3 + \frac{3\eta^4}{4} - \frac{\eta^5}{5}\right)\right]_0^1$$

$$\frac{\delta_1}{\delta} = \frac{3}{10} - \frac{N}{120}$$

By definition, $\delta_2 = \int_0^\infty u/U_\infty(1 - u/U_\infty)dy$. Using a similar substitution for the term dy as was used for δ_1, yields:

$$\frac{\delta_2}{\delta} = \int_0^1 \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) d\eta$$

Finally, after some tedious algebra and integration:

$$\frac{\delta_2}{\delta} = \frac{1}{63}\left(\frac{37}{5} - \frac{N}{15} - \frac{N^2}{144}\right)$$

APPENDIX B

Solution of the Momentum Thickness Equation

In finding the value of δ_2/δ the term u/U_∞ is squared and integrated. From Chapter 3, Equation 33, the relation is:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = \frac{1}{1260}\Big[a(420a + 630b + 504c)$$
$$+ b(420c + 252b) + c(180c)$$
$$+ d(140d + 420a + 360b + 315c)\Big] \qquad (1)$$

The terms a, b, c, and d have been derived in Chapter 3 and are defined by Equations 23, 22, 21, and 24 respectively. To help organize the mathematical calculations, Equation 1 is subdivided into four parts.

Part One

Part One solves the following expression:

$$a(420a + 630b + 504c) \qquad (2)$$

The terms of Part One are expanded as:

$420a = (10080 + 2520N + 420MN)/D$

$630b = (-5670N + 7560M)/D$

$504c = (-1512MN + 2016M^2)/D$

where $M = v_o\delta/\nu$, $N = (\delta^2/\nu)(dU_\infty/dx)$, and

$D = 18 + 6M + M^2$

Substituting the expanded terms into Expression 2 gives:

$$[\ 241920 - 7560N - 26208MN + 181440M + 48384M^2 + 60480N - 18900N^2 - 6552MN^2 + 45360MN + 12096M^2N + 10080MN - 3150MN^2 + 1092M^2N^2 + 7560M^2N + 2016M^3N\ ] / D^2$$

Expression 2 is further reduced by combining similar terms to give:

$$[\ 241920 - 15120N + 29232MN + 181440M + 48384M^2 - 18900N^2 - 9702MN^2 + 19656M^2N - 1092M^2N^2 + 2016M^3N\ ] / D^2 \qquad (3)$$

<u>Part Two</u>

Part Two solves the following expression:

$$b(420c + 252b) \qquad (4)$$

Expanding the terms of Expression 4 gives:

$$420c = (-1260MN + 1680M^2) /D \qquad (5)$$

$$252b = (-2268N + 3024M) /D \qquad (6)$$

Substituting Equations 5 and 6 into Expression 4 yields the result of Part Two which is:

$$[\ 11340MN^2 - 30240M^2N + 20412N^2 - 54432MN + 362886M^2 + 20160M^3\ ] / D^2 \qquad (7)$$

<u>Part Three</u>

Part Three solves the expression:

$c(180c)$

After expanding and combining similar terms, the solution for Part Three becomes:

$$[\ 1620M^2N^2 - 4320M^3N + 2880M^4\ ] / D^2 \tag{8}$$

Part Four

Part Four is the expansion of the expression:

$$d(140d + 420a + 360b + 315c) \tag{9}$$

Taking each term from Expression 9 and expanding it gives:

$$140d = (-180 + 420N + 280MN - 840M - 420M^2) / D \tag{10}$$

$$420a = (10080 + 2520N + 420MN) / D \tag{11}$$

$$360b = (-3240N + 4320M) / D \tag{12}$$

$$315c = (-945MN + 1260M^2) / D \tag{13}$$

Substituting Equations 10, 11, 12, and 13 into Expression 9 yields :

$$\begin{aligned}[\ & -55440 + 1800N + 1470MN - 20880M - 5040M^2 + 27720N \\ & - 900N^2 - 735MN^2 + 10440MN + 2520M^2N + 18480MN \\ & - 490M^2N^2 + 6960M^2N + 1680M^3N - 55440M + 1800MN \\ & + 1470M^2N - 20880M^2 - 5040M^3 - 27720M^2 + 900M^2N \\ & - 600MN^2 + 735M^3N - 10440M^3 - 2520M^4\ ] / D^2\end{aligned}$$

After combining all common terms, the expression of Part Four is:

$$\begin{aligned}[\ & -55440 + 29520N + 32190MN - 76320M - 53640M^2 - 900N^2 \\ & - 1335MN^2 + 11850M^2N - 490M^2N^2 + 2415M^3N - 15480M^3 \\ & - 2520M^4\ ] / D^2\end{aligned} \tag{14}$$

Adding Expressions 3, 7, 8, and 14 and then multiplying them by (1 /1260) gives the expanded version of Equation 1:

$$\int_0^1 \left(\frac{u}{U_\infty}\right)^2 d\eta = [\ 186480 + 14400N + 6990MN + 105120M$$
$$+ 31032M^2 + 612N^2 + 303MN^2$$
$$+ 30M^2N^2 + 111M^3N + 4680M^3$$
$$+ 1266M^2N + 360M^4\ ]\ /\ 1260D \qquad (15)$$

By definition the momentum thickness equation is:

$$\frac{\delta_2}{\delta} = \int_0^1 \left(\frac{u}{U_\infty} - \left(\frac{u}{U_\infty}\right)^2 \right) d\eta$$

Since Equation 15 from above and Equation 32 from Chapter 3 are terms in the momentum thickness equation, the new equation for δ_2 / δ with suction and pressure gradient is:

$$\frac{\delta_2}{\delta} = \frac{(648 + 36N + 9MN + 168M + 24M^2)}{60\ (18 + 6M + M\ )}$$
$$- [\ (186480 + 14400N + 6990MN + 105120M + 31032M^2$$
$$+ 612N^2 + 303MN^2 + 1266M^2N + 30M^2N^2 + 111M^3N$$
$$+ 4680M^3 + 360M^4)\ /\ (1260(18 + 6M + M^2)^2)\ ]$$

After combining similar terms, the non-dimensional form for the momentum thickness equation becomes:

$$\frac{S_2}{S} = (58464 - 729N + 940MN + 40032M + 12816M^2 - 612N^2$$

$$- 303MN^2 + 624M^2N - 30M^2N^2 + 70M^3N + 1872M^3$$

$$+ 144M^4) / (1260(18 + 6M + M^2)^2) \qquad (16)$$

APPENDIX C

Derivation of Shear Stress

To find separation pressure gradient, Pohlhausen first derived the shear stress equation and set it equal to zero. To have separation the shear stress must equal zero. The shear stress equation is:

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y} \right)_0 = \mu \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y} \qquad \text{(note: } \partial \eta / \partial y = 1 / \delta\text{)}$$

Taking Equation 11 from Appendix A and substituting it into the shear stress equation gives:

$$\tau_0 = \frac{\mu}{\delta} \frac{\partial}{\partial \eta} \left[U_\infty (2\eta - 2\eta^3 + \eta^4) + \frac{N}{6} (\eta - 3\eta^2 + \eta^3 - \eta^4) \right] \Bigg|_0$$

The above equation reduces to:

$$\tau_0 = \frac{\mu}{\delta} U_\infty \left(2 + \frac{N}{6} \right) \qquad (1)$$

Rearranging Equation 1 gives:

$$\frac{\tau_0 \delta}{\mu U_\infty} = 2 + \frac{N}{6} \qquad (2)$$

Equation 2 is Pohlhausen's solution for shear stress in terms of pressure gradient only. Equation 26 from Chapter 3 shows the shear stress as:

$$\frac{\tau_0 \delta}{\mu U_\infty} = \frac{24 + 6N + MN}{18 + 6M + M^2} \tag{3}$$

Equation 3 is a function of both pressure gradient and suction. Pohlhausen's solution is a function of only pressure gradient. To find the pressure gradient at separation where the shear stress is zero, Equations 2 and 3 are used. These two equations each equal zero at separation, therefore their pressure gradients at separation can be found.

NOTES

[1] J. Grimson, Advanced Fluid Dynamics and Heat Transfer, (New York: McGraw-Hill, 1971), pp. 145-146.

[2] Grimson, p. 146.

[3] Grimson, p. 145.

[4] J. Kestin, trans., Boundary-Layer Theory, by Hermann Schlichting, 6th rev. ed. (New York: McGraw-Hill, 1968), p. 146.

[5] G.V. Lachmann, ed., Boundary Layer and Flow Control, II (New York: Pergamon Press, 1961), pp. 803-804.

[6] Paul T. Torda, "Boundary Layer Control by Continuous Surface Suction or Injection," Journal of Mathematics and Physics, 31 (1952), p. 208.

[7] Rudolf Iglish, Exact Calculation of Laminar Boundary Layer in Longitudinal Flow Over a Flat Plate with Homogeneous Suction, U.S., National Advisory Committee for Aeronautics, Technical Memorandum No. 1205 (Washington, D.C.: Government Printing Office, April 1949) p. 86.

[8] Kestin, p. 369

[9] Kestin, p. 206.

[10] Robert S. Brodkey, The Phenomena of Fluid Motions, (Reading, Massachusetts: Addison-Wesley, 1967), p. 152.

[11] Grimson, p. 158.

[12] Lachmann, p. 1891.

[13] Lachmann, p. 1893.

[14] Torda, p. 289.

[15] Torda, p. 288.

[16] Kestin, p. 376.

[17] J. H. Preston, The Boundary-layer Flow Over a Permeable Surface through which Suction is Applied, Aeronautical Research Council, Reports and Memoranda No. 2244 (London: His Majesty's Stationery Office, Febr. 1946), p. 12.

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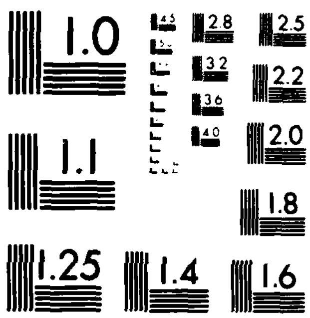

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